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DYNAMICAL STABILITY AND MANEUVERABILITY
OF
DYNAMICALLY UNSTABLE SHIP

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Monterey, California

THESIS

DYNAMICAL STABILITY AND MANEUVERABILITY OF DYNAMICALLY UNSTABLE SHIPS

by

Dae-Song Park

Thesis Advisors:
C. J. Garrison and G. J. Thaler

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Dynamical Stability and Maneuverability
of
Dynamically Unstable Ship

by

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ABSTRACT

The factors affecting the stability and the maneuverability of a dynamically unstable ship were studied using the linear and non-linear equations of the motion of the ship. The DSL/360 Language was used to simulate the dynamics and to study both unstable motions and standard maneuvers.

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TABLE OF SYMBOLS

a_{eff} = effective aspect ratio

A_f = area of movable fin

A_R = area of rudder

C_D = drag coefficient

C_L = lift coefficient

O = origin of reference axes fixed in the ship

$\otimes$ = midship section

x,y,z = system of reference axes fixed in the ship

x_o, y_o, z_o = system of reference axes fixed in the earth

x_G, y_G, z_G = components of distance of C.G. of the ship along x, y and z-axis, respectively.

x_f = distance from O to center of pressure of fin along x-axis.

k_1, k_2, k' = Lamb's coefficients (see ref. [1])

k_1, k_2 = coefficients of rudder deflection rate.

I_x, I_y, I_z = mass moments of inertia of a ship about x,y and z-axis, respectively.

m_2' = lateral added mass, non-dimensionalized.

m_x' = actual plus added mass in the longitudinal direction non-dimensionalized.

m = mass of ship

m_x' = rotational added mass, non-dimensionalized.

ϕ = roll angle

θ = pitch angle

ψ = yaw angle

β = angle of attack in yaw on the hull (or drift angle).

δ = angular deflection of rudder

U = resultant velocity of the fluid relative to a control surface or a fixed fin.

p,q,r = components of resultant angular velocity of the ship about the x, y and z-axis, respectively (= $\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$).

u,v,w, = components of U along the x, y and z-axis, respectively.

D = drag force

F = total force

L = lift force or length of ship (LBP)

X,Y,Z = components of resultant total force acting at the origin O directed along x,y and z-axis, respectively.

X_o, Y_o, Z_o = values of X,Y and Z at initial equilibrium condition.

$\vec{H}$ = angular momentum

H = draft of ship

$\vec{\Omega}$ = angular velocity

$\vec{\mathcal{M}}$ = resultant total moments

K,M,N = components of a resultant total moment acting on a ship about the x,y and z-axis, respectively.

Y_v = partial derivative of Y with respect to v

$Y_{\dot{v}}$ = " " $\dot{v}$

Y_r = " " r

$Y_{\dot{r}}$ = " " $\dot{r}$

Y_δ = " " δ

N_v = partial derivative of N with respect to v

$N_{\dot{v}}$ = " " $\dot{v}$

N_r = " " r

$N_{\dot{r}}$ = " " $\dot{r}$

N_δ = " " δ

Note:

1) Symbol of prime ('), except where otherwise stated indicates the values of an item non-dimensionalized.
2) Symbol of dot (·) indicates differential value of an item with respect to time.
3) Subscript f indicates the value of an item of fin.
4) Subscript h indicates the value of an item of ship's hull.
5) Subscript f_1 indicates the value of an item of the large deadwood included rudder.
6) Subscript f_2 indicates the value of an item of the small deadwood or rudder only.

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I. INTRODUCTION

A ship is said to be dynamically stable on a straight course or in a turn of constant curvature if, when slightly disturbed from it's steady motion, it will soon resume that same motion along a slightly shifted path without any correcting control being applied. A ship which is dynamically unstable in straight line motion cannot maintain straight line motion when there is no control. However, with regard to maneuverability, a ship which is too stable will not turn as tightly as a somewhat less stable ship so that a highly stable ship may compromise maneuverability. Therefore, a ship should be designed for a moderate amount of stability so as to be able to go straight without compromising maneuverability too much.

With these factors in mind then, the stability of five different ships was studied in Part III and maneuverability in Part IV. The first objective of this thesis was to investigate the stability of a ship with movable fin. The second objective was to investigate the maneuverability of an unstable ship in standard maneuvers to compare with those of a stable ship. A third objective was to apply DSL/360 Language to simulate the ship motions.

II. EQUATION OF SHIP MOTION IN THE HORIZONTAL PLANE

As developed in Ref. 2, the general motion of a ship is that of a rigid body in six degrees of freedom subject to gravity and buoyancy forces as well as controlling forces and hydrodynamic reactions and subject to hydrodynamic or other disturbances or excitations.

Considering axes fixed to the body and parallel with the principal axes of inertia but with arbitrary origin as shown in Fig. 1, the linear and angular momentum equations may be written as

$$\begin{aligned} \vec{F} &= \frac{d}{dt}(m\vec{U}) = m\dot{\vec{U}} + \vec{\Omega}\times(m\vec{U}) \\ \vec{\mathcal{M}} &= \frac{d}{dt}(\vec{H}) = \dot{\vec{H}} + \vec{\Omega}\times\vec{H} \end{aligned} \tag{1}$$

where

m = mass of ship

$\vec{U}$ = linear velocity vector

= $u\vec{i} + v\vec{j} + w\vec{k}$

u = rate of surging

v = rate of swaying

r = rate of heaving

= angular velocity vector

= $p\vec{i} + q\vec{j} + r\vec{k}$ (2)

p = rate of roll = ϕ

q = rate of yaw = θ

r = rate of pitch = ψ

$\vec{F}$ = force vector acting on the body

$$= X\vec{i} + Y\vec{j} + Z\vec{k} \tag{3}$$

X = hydrodynamic force on body along x-axis

Y = hydrodynamic force on body along y-axis

Z = hydrodynamic force on body along z-axis

$\vec{\mathcal{M}}$ = moment vector acting on body about a fixed point

$$= K\vec{i} + M\vec{j} + N\vec{k} \tag{4}$$

K = rolling moment about x-axis

M = pitching moment about the y-axis

N = yawing moment about the z-axis

H = angular moment vector

$$= I_x p\vec{i} + I_y q\vec{j} + I_z r\vec{k} \tag{5}$$

I_x, I_y, I_z = mass moment of inertia about the axis x, y and z respectively.

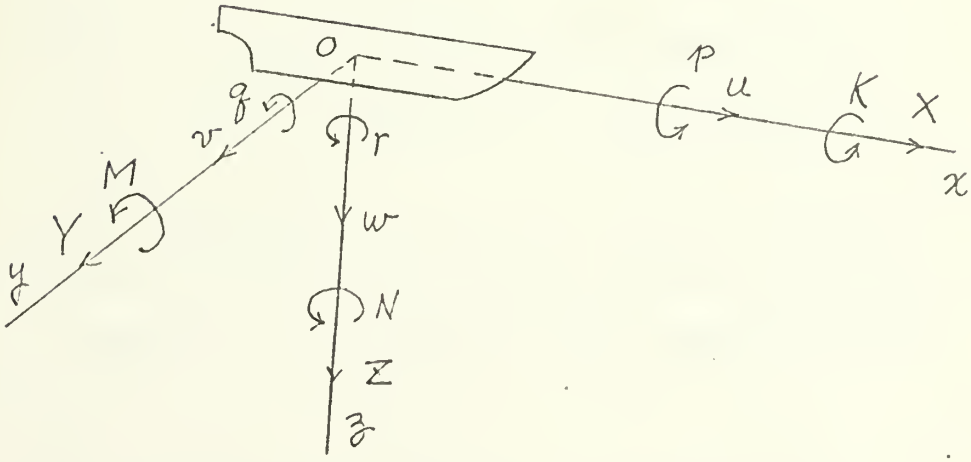


FIGURE 1. Body axes

Inserting Equations (2), (3), (4) and (5) in Eq. (1) gives:

$$\left.\begin{aligned} X &= m\{\dot{u} + qw - rv - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})\} \\ Y &= m\{\dot{v} + ru - pw - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})\} \\ Z &= m\{\dot{w} + pv - qu - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})\} \end{aligned}\right\} \quad (6)$$

$$\left.\begin{aligned} K &= I_x\dot{p} + (I_z - I_y)qr + m\{y_G(\dot{w} + pv - qu) - z_G(\dot{v} + ru - pw)\} \\ M &= I_y\dot{q} + (I_x - I_z)rp + m\{z_G(\dot{u} + qw - rv) - x_G(\dot{w} + pv - qu)\} \\ N &= I_z\dot{r} + (I_y - I_x)pq + m\{x_G(\dot{v} + ru - pw) - y_G(\dot{u} + qw - rv)\} \end{aligned}\right\} \quad (7)$$

In Eq. (6) and (7), the left hand side represents the forces and moments acting on the ship and the right hand side represents the dynamic responses of the ship, respectively. The forces and moments acting on the ship depend on the geometry of the hull, the motion and the fluid. That is,

$\vec{F}$ (or $\vec{m}$) = f (properties of ship, properties of fluid, properties of ship motion)

Since the properties of ship and fluid are constant for a given ship in a given fluid without excitation forces, the forces or moments may be expressed as a function of the motion as,

$$\vec{F}(\text{or } \vec{m}) = f(x_0, y_0, z_0, \phi, \theta, \psi, u, v, w, p, q, r, \dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}, \delta, \dot{\delta}, \ddot{\delta}, \text{etc}) \qquad (8)$$

where x_o, y_o and z_o represent a system of reference axes fixed in the earth and

$x_o, y_o, z_o, \phi, \theta, \psi$ = orientation parameters

$\left.\begin{matrix} u, v, w, p, q, r \\ \dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r} \end{matrix}\right\}$ = motion parameters

$\delta, \dot{\delta}, \ddot{\delta}$ = control surface parameters

Considering motion in unrestricted calm water, only three degrees of freedom are of concern: yaw, sway and surge. Therefore, the forces and moments are independent of the change in orientation with respect to the earth axes.

Furthermore since the forces and moments produced on the ship as a result of $\dot{\delta}$ and $\ddot{\delta}$ are normally negligible.

$$\vec{F}(\text{or } \vec{m}) = f(u, v, r, u, v, r, \delta) \tag{9}$$

Since the motion is restrained to the horizontal plane for the surface ship in calm water as prescribed in the previous section, roll, pitching and heaving are taken as zero, i.e., p=q=w=0 in Eqs. (6) and (7). Furthermore, most ships have $y_G = 0$ so that Eqs. (6) and (7) may be written as,

$$\begin{aligned} X &= m(\dot{u} - rv - x_G r^2) \\ Y &= m(\dot{v} + ru + x_G \dot{r}) \\ N &= I_z \dot{r} + m\{x_G(\dot{v} + ur)\} \end{aligned} \tag{10}$$

From Eqs. (9) and (10) then, these become:

$$\begin{aligned} &\mathrm{X:} \quad m(\dot{u} - rv - x_G r^2) = f_X(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta) \\ &\mathrm{Y:} \quad m(\dot{v} + ru + x_G \dot{r}) = f_Y(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta) \qquad (11) \\ &\mathrm{N:} \quad I_z \dot{r} + m x_G (\dot{v} + ur) = f_N(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta) \end{aligned}$$

III. DYNAMIC STABILITY OF THE SHIP

Dynamic stability is directly related to the magnitude of yaw and sway deviations caused by small initial disturbances. In many problems related to dynamic stability with small disturbances, the linear analysis of motion is quite useful and will be applied in the following to the present problem.

A. LINEAR EQUATION OF SHIP MOTION

To simplify the equation of ship motion, the following assumptions are made:

(1) The ship has adequate stability.

(2) The water is so calm that the rolling, pitching and heaving motions are negligible. Therefore, the motion of ship may be regared as taking place in the horizontal plane only.

(3) Yaw and sway do not affect the forward speed appreciably, and small changes in forward speed do not affect yaw and sway motions so that the surge equation is decoupled.

(4) The ship has one rudder at the stern along the centerline and the effect of the propeller is disregarded.

(5) The forces and moments on hull and rudder are expressed by means of velocity and acceleration derivatives.

In order to linearize the equations of ship motion about an initial equilibrium condition of motion with the previous assumptions, only the linear terms in the change of the value

of the variables from equilibrium, i.e. Δu, Δv, Δr, $\Delta \dot{u}$, $\Delta \dot{v}$, $\Delta \dot{r}$ and $\Delta\delta$ are considered.

From Eq. (9) then,

$$X = X_o + X_u \Delta u + X_v \Delta v + X_r \Delta r + X_{\dot{u}} \Delta \dot{u} + X_{\dot{v}} \Delta \dot{v} + X_\delta \Delta \delta$$

Since in initial equilibrium $v_o = r_o = \dot{u}_o = \dot{v}_o = \dot{r}_o = \delta_o = 0$,

$$\Delta v = v - v_o = v$$
$$\Delta r = r - r_o = r$$
$$\Delta \dot{u} = \dot{u} - \dot{u}_o = \dot{u}$$
$$\Delta \dot{v} = \dot{v} - \dot{v}_o = \dot{v}$$
$$\Delta \dot{r} = \dot{r} - \dot{r}_o = \dot{r}$$
$$\Delta \delta = \delta - \delta_o = \delta$$

Therefore,

$$X = X_o + X_u \Delta u + X_v v + X_r r + X_{\dot{u}} \dot{u} + X_{\dot{v}} \dot{v} + X_{\dot{r}} \dot{r} + X_\delta \delta \qquad (12)$$

and likewise,

$$Y = Y_o + Y_u \Delta u + Y_v v + Y_r r + Y_{\dot{u}} \dot{u} + Y_{\dot{v}} \dot{v} + Y_{\dot{r}} \dot{r} + Y_\delta \delta \qquad (13)$$
$$N = N_o + N_u \Delta u + N_v v + N_r r + N_{\dot{u}} \dot{u} + N_{\dot{v}} \dot{v} + N_{\dot{r}} \dot{r} + N_\delta \delta$$

where X_o, Y_o and N_o are the values of the initial equilibrium conditions.

In the initial equiliburium conditions of straight ahead motion at constant speed, there are no forces acting on the ship. Hence

$$X_o = Y_o = N_o = 0$$

Furthermore, since the ship is symmetric about the x-z plane

$$Y_u = Y_{\dot{u}} = X_v = X_r = X_{\dot{r}} = N_u = N_{\dot{u}} = 0$$

and

$$X_\delta = 0$$

Therefore,

$$\begin{aligned} X &= X_u \Delta u + X_{\dot{u}} \dot{u} \\ Y &= Y_v v + Y_r r + Y_{\dot{v}} \dot{v} + Y_{\dot{r}} \dot{r} + Y_\delta \delta \\ N &= N_v v + N_r r + N_{\dot{v}} \dot{v} + N_{\dot{r}} \dot{r} + N_\delta \delta \end{aligned} \tag{14}$$

Linearizing Eq. (10) gives

$$\begin{aligned} X &= m\left\{(\dot{u}_0 + \Delta u) - (r_0 + \Delta r)(v_0 + \Delta v) - x_G (r_0 + \Delta r)^2\right\} \\ &\doteq m(\Delta \dot{u} - \Delta r \Delta v - x_G \Delta r^2) \end{aligned}$$

This is simplified further by neglecting second order terms so that

$$X = m \Delta u = m \dot{u}$$

and likewise,

$$\begin{aligned} Y &= m(\dot{v} + r u_0 + x_G \dot{r}) \\ N &= I_z \dot{r} + m x_G (\dot{v} + r u_0) \end{aligned} \tag{15}$$

From Eqs. (14) and (15)

$$\begin{aligned} &(X_{\dot{u}} - m)\dot{u} + X_u \Delta u = 0 \\ &(Y_{\dot{v}} - m)\dot{v} + Y_v v + (Y_{\dot{r}} - m x_G)\dot{r} + (Y_r - m u_0) r + Y_\delta \delta = 0 \\ &(N_{\dot{v}} - m x_G)\dot{v} + N_v v + (N_{\dot{r}} - I_z)\dot{r} + (N_r - m x_G u_0) r + N_\delta \delta = 0 \end{aligned} \tag{16}$$

For convenience, Eq. (16) is non-dimensionalized. Every term of the first and second equations for the X-force and the Y-force in Eq. (16) has the dimensions of force and the third equation for moments has the dimension of a moment.

To non-dimensionalize these, the force equations are divided through by $(\frac{\rho}{2}LHU^2)$ and the moment equation is divided by $(\frac{\rho}{2}L^2HU^2)$ where ρ = dinsity of ship, L = ship's length, U = ship speed and H = draft of ship. This gives,

$$(X'_{\dot u}-m')\dot u'+X'_u\Delta u=0$$

$$(Y'_{\dot v}-m')\dot v'+Y'_v v'+(Y'_{\dot r}-m'x'_G)\dot r'+(Y'_r-m'u'_0)r'+Y'_\delta\delta \qquad (17)$$

$$(N'_{\dot v}-m'x'_G)\dot v'+N'_v v'+(N'_{\dot r}-I'_z)\dot r'+(N'_r-m'x'_G u'_0)r'+N'_\delta\delta$$

where

$$m'=m\Big/\frac{\rho}{2}L^2H\ ,\quad v'=v/U\ ,\quad \dot v'=v\Big/\frac{U^2}{L}$$

$$u'_0=u_0/U\approx 1$$ for small perturbations,

$$I'_z=I_z\Big/\frac{\rho}{2}L^4H\ ,\quad r'=r\Big/\frac{U}{L}\ ,\quad \dot r'=\dot r L^2/U^2$$

$$x'_G=x_G/L\ ,\quad Y'_\delta=Y_\delta\Big/\frac{\rho}{2}LHU^2\ ,\quad N'_\delta=N_\delta\Big/\frac{\rho}{2}L^2HU^2,$$

$$Y'_v=Y_v\Big/\frac{\rho}{2}LHU\ ,\quad Y'_r=Y_r\Big/\frac{\rho}{2}L^2HU\ ,\quad N'_v=N_v\Big/\frac{\rho}{2}L^2HU\ ,$$

$$N'_r=N_r\Big/\frac{\rho}{2}L^3HU\ ,\quad Y'_{\dot v}=Y_{\dot v}\Big/\frac{\rho}{2}L^2H\ ,\quad N'_{\dot v}=N_{\dot v}\Big/\frac{\rho}{2}L^3H$$

$$N'_{\dot r}=N_{\dot r}\Big/\frac{\rho}{2}L^4H$$

In Eq. (17) the dimensionless time is defined as

$$t' = \frac{U}{L} t$$

B. DYNAMIC STABILITY WITH FIXED CONTROL

Taking the Laplace Transform of Eq. (17) with the rudder fixed at $\delta = 0$ gives:

$$\begin{aligned} &[(Y'_{\dot v} - m')s + Y'_v]v'(s) + [(Y'_{\dot r} - m'x'_G)s + (Y'_r - m'u'_0)]r'(s) = 0 \\ &[(N'_{\dot v} - m'x'_G)s + N'_v]v'(s) + [(N'_r - I'_z)s + (N'_r - m x'_G u'_0)]r'(s) = 0 \end{aligned} \qquad (18)$$

Then, using Cramer's rule

$$v'(s) = \frac{0}{\Delta}$$

$$r'(s) = \frac{0}{\Delta}$$

where

$$\Delta = \begin{vmatrix} (Y'_{\dot v} - m')s + Y'_v & (Y'_{\dot r} - m'x'_G)s + (Y'_r - m') \\ (N'_{\dot v} - m'x'_G)s + N'_v & (N'_{\dot r} - I'_z)s + (N'_r - m'x'_G) \end{vmatrix}$$

Expanding this determinant gives the characteristic equation

$$\Delta = A s^2 + B s + C \qquad (19)$$

where

$$A = (Y'_{\dot{v}} - m')(N'_{\dot{r}} - I_z) - (Y'_{\dot{r}} - m'x'_G)(N'_{\dot{v}} - m'x'_G)$$

$$\begin{aligned} B = {} & (Y'_{\dot{v}} - m')(N'_{\dot{r}} - m'x'_G) - Y'_v(N'_{\dot{r}} - I'_z) - \\ & N'_v(Y'_{\dot{r}} - m'x'_G) - (Y'_r - m')(N'_{\dot{v}} - m'x'_G) \end{aligned} \qquad (20)$$

$$C = Y'_v(N'_r - m'x'_G) - N'_v(Y'_r - m')$$

The roots of Eq. (19) are

$$\sigma_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A} = \frac{-\frac{B}{A} + \sqrt{(\frac{B}{A})^2 - 4(\frac{C}{A})}}{2} \qquad (21)$$

$$\sigma_2 = \frac{-\frac{B}{A} - \sqrt{(\frac{B}{A})^2 - 4(\frac{C}{A})}}{2}$$

Therefore, the free responses of ship, i.e., sway and yaw motions are given by

$$\begin{aligned} v'(t') &= v_1 e^{\sigma_1 t'} + v_2 e^{\sigma_2 t'} \\ r'(t') &= r_1 e^{\sigma_1 t'} + r_2 e^{\sigma_2 t'} \end{aligned} \qquad (22)$$

where v_1, v_2, r_1 and r_2 are arbitrary constants depending on initial conditions.

As shown in Eq. (22), the dynamical stability depends on the roots of the characteristic equation. Since $\sigma_1 \geqslant \sigma_2$,

$$\lim_{t' \to \infty} v'(t') = 0 \quad \text{and} \quad \lim_{t' \to \infty} r'(t') = 0 .$$

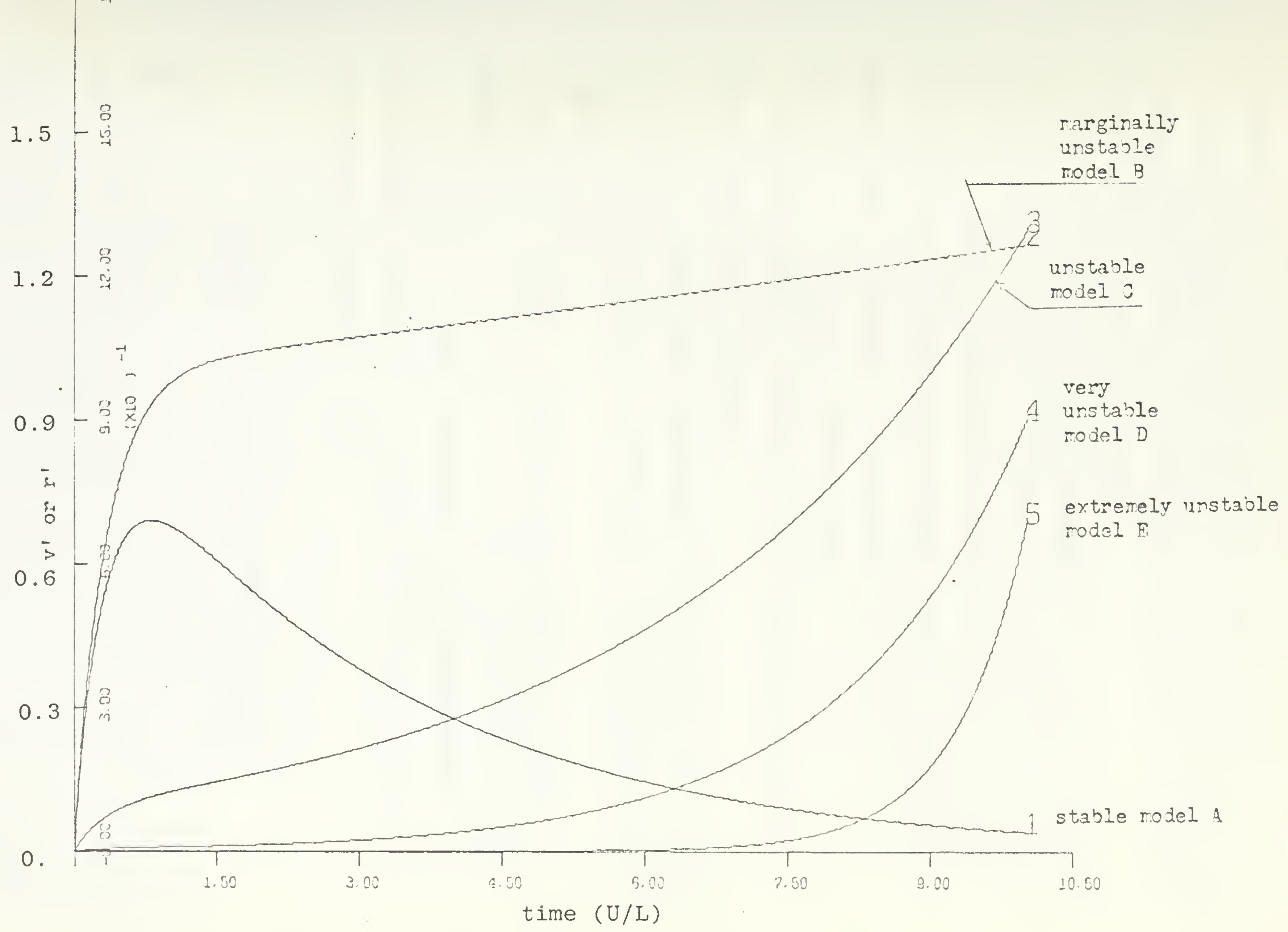


FIGURE 2. Response of the yaw rate and sway velocity to

if and only if $\sigma_1 \leq 0$

If $\sigma_1 > 0$ then, $v'(\infty) \neq 0$ and $r'(\infty) \neq 0$ which means that the ship is dynamically unstable in straight line motion. The root, σ_1 is called the 'stability index'.

In Appendix B and computer program 1, a five foot Series 60 model of the Davidson Laboratory with block coefficient, $C_B = 0.7$, and no propeller, was modified to be (a) stable (b) marginally unstable (c) unstable (d) very unstable and (e) extremely unstable. These models are referred to as model A, B, C, D and E, respectively.

The hydrodynamic derivatives and stability indices of these models are in Table 1.

Eqs. (21) and (22) were simulated to obtain the response of each model to small a disturbance. (See computer program 7 and Fig. 2)

As shown in Fig. 2 (which is the result of computer program 7), the yaw rate, r, as well as the sway velocity, v, of the stable model A decreases as time increases but the unstable models B, C, D and E tend to diverge due to the positive stability index.

TABLE I. HYDRODYNAMIC DERIVATIVES AND STABILITY INDICES OF THE FIVE MODELS.

model	Y_v'	N_v'	Y_r'	N_r'	$Y_{\dot v}'$	$N_{\dot v}'$	$Y_{\dot r}'$	$N_{\dot r}'$	σ_1
A	-.3544	-.0938	+.0774	-.0635	-.1880	+.0040	+.0040	-.0132	-.3213
B	-.3131	-.1145	+.0567	-.0532	-.1868	+.0034	+.0034	-.0129	+.0238
C	-.3131	-.1266	+.0446	-.0435	-.1868	+.0024	+.0024	-.0121	+.2561
D	-.3131	-.1447	+.0265	-.0380	-.1868	+.0010	+.0010	-.0117	+.5194
E	-.1867	-.1777	-.0065	-.0216	-.1800	0.	0.	-.0112	+1.3438

C. STABILIZATION OF THE DYNAMICALLY UNSTABLE SHIP WITH FIXED RUDDER

As described in the previous section, a dynamically unstable ship has

$$\sigma_1 = \frac{-\frac{B}{A} + \sqrt{(\frac{B}{A})^2 - 4(\frac{C}{A})}}{2} > 0$$

To stabilize this unstable ship, σ_1 must be made less than or equal to zero, i.e., $\sigma_1 \leq 0$ or,

$$\sqrt{(\frac{B}{A})^2 + 4(\frac{C}{A})} \leq \frac{B}{A}$$

By the nature of hydrodynamic derivatives for the ship $A > 0$ and $B > 0$ always. Therefore, C must be greater than or equal to zero for stability. Thus, C is called the "stability criterion."

From Eq. (20)

$$C = Y_v'(N_r' - m'x_G') - N_v'(Y_r' - m') \geq 0 \qquad (20a)$$

Then, considering the characteristics of each of the hydrodynamic derivative, mass and the distance of the C.G. from the origin of the fixed coordinates on the ship.

Y_v is always large negative quantity

N_r is always large negative quantity

N_v may be positive or negative

Y_r is small but may be positive or negative

m is always large and positive

X_G is always small but may be positive or negative

Finally, N_v has the greatest effect on dynamical stability since the term $(Y_r' - m')$ is a large negative quantity in equation (20a). If, for example, N_v becomes more positive, the stability criterion C will become more positive so that the stability of the ship will be improved. It is noted that N_v can be controlled by a lifting surface.

1. Sensitivity of Dynamic Stability to a Lifting Surface

Equation (21) was simulated using the computer program DSL/360 Language for the effect of the location, size and aspect ratio of a lifting surface (movable fin) on the models A, B, C, D and E. (See the computer program 2). The results are summarized in Table II and III.

The following are indicated by Tables II and III and by Figs. 3, 4 and 5:

1) The position of the fin at which the stability index is maximum is independent of the area and aspect ratio of the fin, because this position is the pivot point at which the relative flow is parallel to the centerline of the ship. (See Fig. 14)

The parallel flow makes no attack angle on the fin so that Y_v, N_v, Y_r and N_r of the fin are zero but $Y_{\dot{v}}$, $N_{\dot{v}}$, $Y_{\dot{r}}$ and $N_{\dot{r}}$ are not zero as described in Appendix A. Therefore, the stability criterion C in Eq. (20) is the same as that of the

TABLE II. EFFECT OF FIN AREA AND FIN LOCATION WITH FIXED ASPECT RATIO ON DYNAMICAL STABILITY

Fin	X_f/L	A	B	C	D	E
No fin		-0.3213	+0.0238	+0.2561	+0.5194	+1.3438
$a_{eff}=1.5$	- 0.5	- .4366	- .1066	+ .1108	+ .3625	+1.1684
$A_f=0.0056$	$\mathfrak{X}$	- .3492	- .0020	- .2324	+ .4968	+1.3223
$h_f=0.015$	pivot point	- .32079	+ .0242	+ .2569	+ .5197	+1.3441
		(0.48)	(0.4)	(0.34)	(1.305)	(0.27)
	+ 0.5	- .3208	+ .0226	+ .2512	+ .5109	+1.3288
$a_{eff}=1.5$	- 0.5	- .5707	- .2616	- .0642	+ .1710	+ .9474
$A_f=.01265$	$\mathfrak{X}$	- .3834	- .0341	+ .2026	+ .4682	+1.2944
$h_f=.0225$	pivot point	- .3203	+ .0246	+ .2567	+ .5199	+1.3442
		(0.48)	(0.4)	(0.34)	(0.305)	(0.27)
	+ 0.5	- .3204	+ .0212	+ .2458	+ .5011	+1.3110
$a_{eff}=1.5$	- 0.5	- .7376	- .4583	- .2887	- .0780	- .6717
$A_f=.0225$	$\mathfrak{X}$	- .4299	- .0780	+ .1617	+ .4286	+1.2548
$h_f=.03$	pivot point	- .3197	+ .0250	+ .2571	+ .5202	+1.3444
		(0.48)	(0.4)	(0.34)	(0.305)	(0.27)
	+ 0.5	- .3199	+ .0196	+ .2393	+ .4893	+1.2888
$a_{eff}=1.5$	- 0.5	-1.0956	- .8930	- .7921	- .6486	- .0573
$A_f=0.0506$	$\mathfrak{X}$	- .5532	- .1963	+ .0507	- .3198	+1.1427
$h_f=0.045$	pivot point	- .3185	+ .0260	+ .2579	+ .5209	+1.3448
		(0.48)	(0.4)	(0.34)	(0.305)	(0.27)
	+ 0.5	- .3188	+ .0162	+ .2259	+ .4643	+1.2393
destabilizing area		0.415L~ 0.5L	0.35L~ 0.445L	0.3L ~ 0.375L	0.27L ~ 0.34L	0.24L~ 0.3L

Note: All values in column A,B,C,D and E (except the values in ( ) which mean pivot point distance from $\mathfrak{X}$) are σ_1 of model A,B,C,D and E, respectively.

TABLE III. EFFECT OF FIN LOCATION AND ASPECT RATIO WITH FIXED AREA ON DYNAMICAL STABILITY

Fin Type	X_f/L	Model A	B	C	D	E
No fin		-0.3213	+0.0238	+0.2561	+0.5194	+1.3438
a_{eff}= 1.5	- 0.5	-0.4366	-0.1066	+0.1108	+0.3625	+1.1684
A_f=0.0056	$\mathfrak{X}$	-0.3492	-0.0020	+0.2324	+0.4968	+1.3223
h_f=0.015	pivot	-0.3208	+0.0242	+0.2569	+0.5197	+1.3441
	point	(0.48)	(0.40)	(0.34)	(0.305)	(0.27)
	+ 0.5	-0.3208	+0.0226	+0.2512	+0.5109	+1.3288
a_{eff}= 2.	- 0.5	-0.4557	-0.1275	+0.0872	+0.3373	+1.1442
A_f=0.0056	$\mathfrak{X}$	-0.3539	-0.0063	+0.2285	+0.4932	+1.3189
h_f=0.0173	pivot	-0.3207	+0.0242	+0.2565	+0.5197	+1.3439
	point	(0.48)	(0.4)	(0.34)	(0.305)	(0.27)
	+ 0.5	+0.3208	+0.0224	+0.2505	+0.0510	+1.3265
a_{eff}= 3.	- 0.5	-0.4823	-0.1572	+0.0543	+0.3021	+1.1036
A_f=0.0056	$\mathfrak{X}$	-0.3605	-0.0122	+0.2231	+0.4881	+1.3164
h_f=0.0212	pivot	-0.3206	+0.0242	+0.2565	+0.5197	+1.3440
	point	(0.48)	(0.40)	(0.34)	(0.305)	(0.27)
	+ 0.5	-0.3207	+0.0221	+0.2495	+0.5078	+1.3235
a_{eff}= 4.	- 0.5	-0.5000	-0.1769	+0.0325	+0.2788	+1.0786
A_f=0.0056	$\mathfrak{X}$	-0.3649	-0.0162	+0.2195	+0.4847	+1.3112
h_f=0.02448	pivot	-0.3206	+0.0244	+0.2565	+0.5198	+1.3441
	point	(0.48)	(0.4)	(0.34)	(0.305)	(0.27)
	+ 0.5	-0.3206	+0.0219	+0.2488	+0.5066	+1.3215

Note: All values in column A,B,C,D and E (except the values in ( ) which mean pivot point distance from $\mathfrak{X}$) are σ_1 of model A,B,C,D and E, respectively.

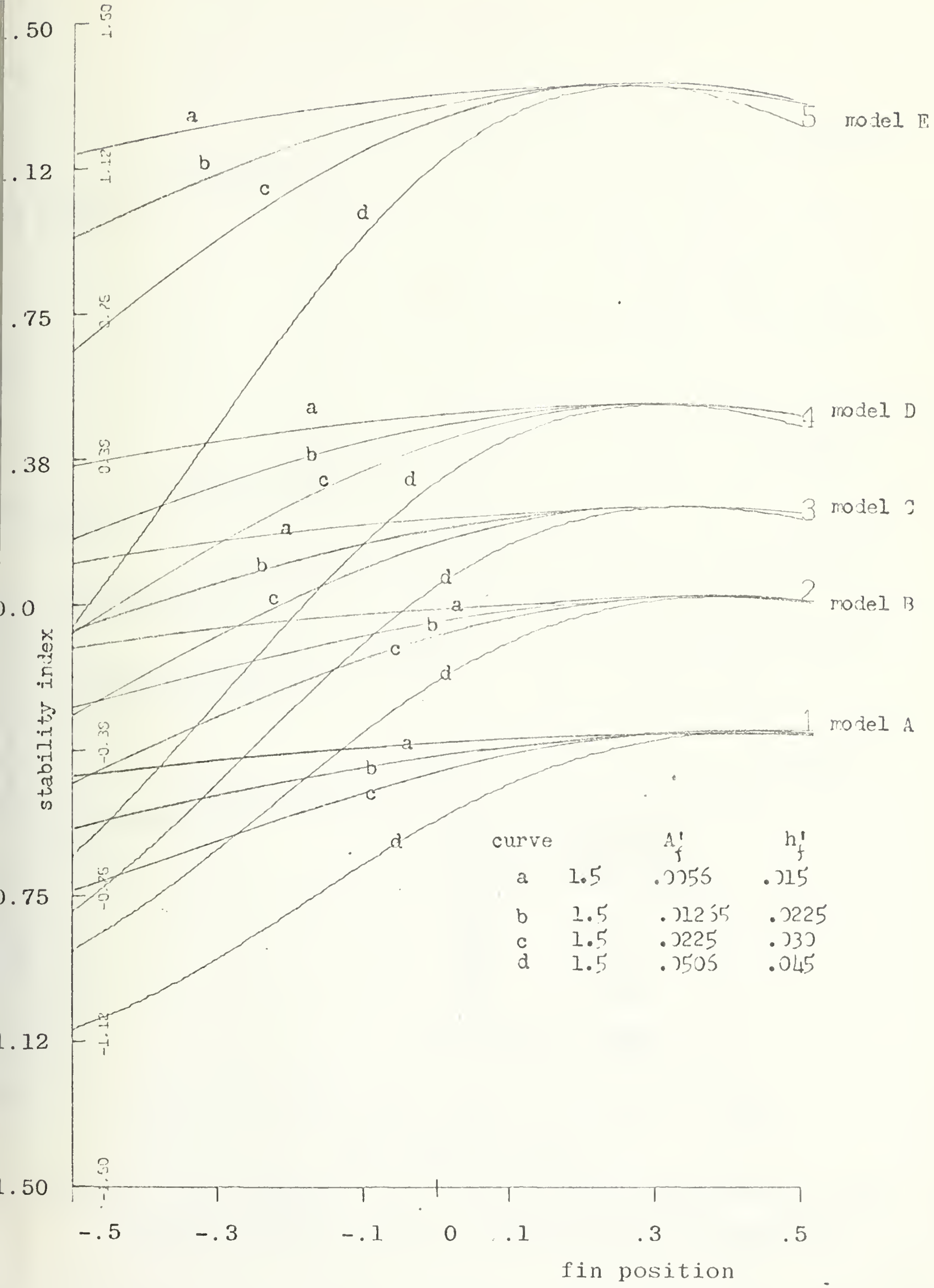


Fig. 3 The effect of fin location and fin area with constant aspect ratio on stability of the ship.

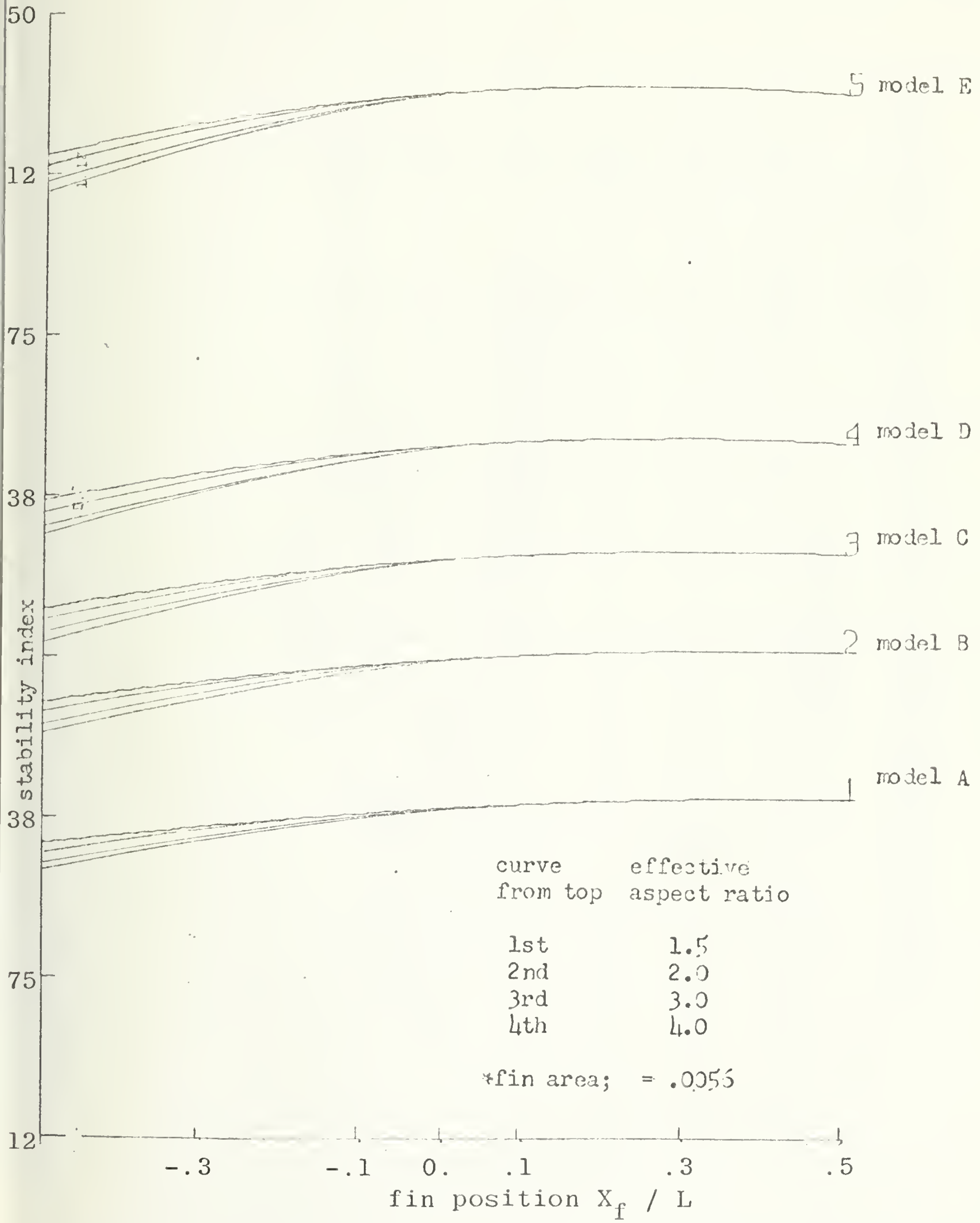


FIGURE 4. The effect of fin location and aspect ratio with constant area of fin on the stability of the ship.

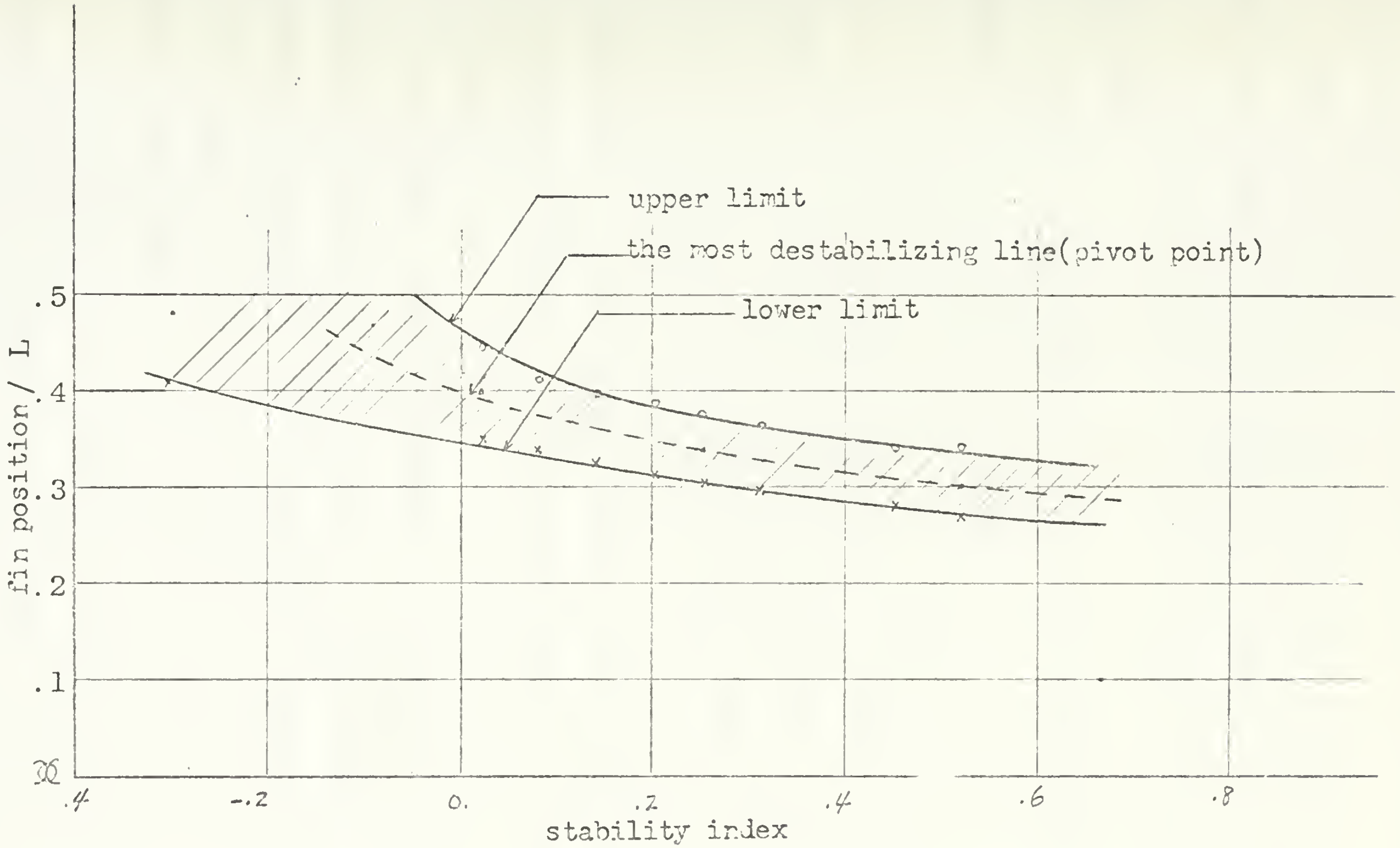


Fig.5 Destabilizing area of the fin and the most destabilizing point

no-fin condition. In other words, the stability criterion, C is minimum at the pivot point during shifting of the fin from stern to bow.

This pivot point is the position of the fin at which it exerts maximum destabilizing influences on the ship.

2) The effect of the fin on stability increases with fin size.

3) The more unstable the ship, the closer the pivot point moves toward the midship.

4) Compared to the no-fin condition, the placement of the fin near the pivot point has a negative effect on stability. This criterion is called the "destabilizing area" in this thesis. This destabilizing area is due to the relative change in A, B, and C in Eq. (21) as the fin location is shifted. C is almost constant near the pivot point and the destabilizing area is independent of the size and type of the fin.

5) Moving the fin toward the stern from midship improves the dynamic stability of the unstable ship. Conversely, moving the fin forward of midship does not cause much change from no-fin condition regardless of the size or type of the fin.

6) Stability is sensitive to the fin area but rather insensitive to changes in the aspect ratio of the fin.

2. Sensitivity of Stability of the Unstable Ship to the Position of the C.G. of the Ship

It is clear that moving the C.G. toward the bow tends to make 'C' more positive in Eq. (20). The effect of moving

C.G. was studied by use of Eqs. (20) and (21) using DSL/360 Language for model C. The results are shown in Table IV. (See computer program 3).

TABLE IV. THE EFFECT OF LOCATION OF C.G. ON THE STABILITY INDEX

$x_{G/L}$	-0.4	-0.25	0	+0.05	+0.1	+0.25
σ_1	+ .6424	+ .4022	+ .0775	+ .015	- .0473	- .2297

As shown in the above table, stability was improved by moving the C.G. toward the bow as expected. However, this is not a practical method of stabilizing an unstable ship.

D. STABILIZATION OF THE DYNAMICALLY UNSTABLE SHIP WITH RUDDER CONTROL

In previous sections, the stabilization of a dynamically unstable ship using lifting surface was discussed. However, it is clear that the dynamically unstable ship can be made directionally stable if the rudder can be forced to compensate for disturbances so that the ship will maintain course with only small oscillations in yaw and sway. If the deflection of the rudder, δ, is proportional to the heading error, ψ, and the angular velocity, $r = \dot{\psi}$, then the equation of rudder deflection becomes

$$\delta(t') = k_1 \psi(t') + k_2 \dot{\psi}(t') \tag{23}$$

where k_1 and k_2 are constants of proportionality of the control system.

Inserting Eq. (23) into Eq. (17) and taking Laplace Transform gives

$$\begin{aligned} &\left[(Y'_{\dot{v}} - m')s + Y'_v\right] v'(s) + \left[(Y'_{\dot{r}} - m'x'_G)s^2 + (Y'_r - m' + Y'_\delta k_2)s + Y'_\delta k_1\right] \psi(s) = 0 \\ &\left[(N'_{\dot{v}} - m'x'_G)s + N_v\right] v'(s) + \left[(N'_{\dot{r}} - I'_z)s^2 + (N'_r - m'x_G' + k_2 N'_\delta)s + k_1 N'_\delta\right] \psi(s) = 0 \end{aligned} \tag{24}$$

As these algebraic equations of (24) are homogeneous, a non-trivial solution ($v \neq \psi \neq 0$) demands the determinant of the system, i.e., characteristic equation, to be zero.

Let the characteristic equation be

$$As^3 + Bs^2 + Cs + D = 0 \tag{25}$$

where

$$A = (Y'_{\dot{v}} - m')(N'_{\dot{r}} - I'_z) - (Y'_{\dot{r}} - m'x'_G)(N'_{\dot{v}} - m'x'_G)$$

$$\begin{aligned} B = {} & (Y'_{\dot{v}} - m')(N'_r - m'x'_G + k_2 N'_\delta) + Y'_v(N'_{\dot{r}} - I'_z) - \\ & N'_v(Y'_{\dot{r}} - m'x'_G) - (Y'_r - m' + k_2 Y'_\delta)(N'_{\dot{v}} - m'x'_G) \end{aligned}$$

$$\begin{aligned} C = {} & (Y'_{\dot{v}} - m')(k_1 N'_\delta) + Y'_v(N'_r - m'x'_G + k_2 N'_\delta) - \\ & N'_v(Y'_r - m' + k_2 Y'_\delta) - k_1 Y'_\delta(N'_{\dot{v}} - m'x'_G) \end{aligned}$$

$$D = k_1 Y'_v N'_\delta - k_1 N'_v Y'_\delta$$

Let the roots of Eq. (25) be σ_1', σ_2' and σ_3'. Then,

$$\begin{aligned} \psi(t') &= \psi_1 e^{\sigma_1' t'} + \psi_2 e^{\sigma_2' t'} + \psi_3 e^{\sigma_3' t'} \\ v'(t') &= v_1 e^{\sigma_1' t'} + v_2 e^{\sigma_2' t'} + v_3 e^{\sigma_3' t'} \end{aligned} \tag{26}$$

If and only if σ_1', σ_2' and σ_3' are negative, then $\psi(\infty)$ and $v'(\infty)$ will be zero and the unstable ship will be stable. Furthermore, if the roots are three repeated roots, i.e., $\sigma_1' = \sigma_2' = \sigma_3' < 0$, the ship will be stable with the shortest possible response.

By designing for the optimum value of k_1 and k_2 in the control mechanism, the ship will be automatically controlled for suitable stability.

To find the three repeated negative roots of the characteristic Eq. (25), the 'MITROVIC METHOD' was applied. (See Appendix C).

In computer program 4, the optimum k_1 and k_2 for the three repeated negative roots of Eq. (25) were calculated for model C and Eq. (26) was simulated by the DSL/360 Language. (See computer program 4).

As shown in Fig. 5-1 which is the result of computer program 4, the response with proper k_1 and k_2 for three negative real roots of characteristic equation (25) is an exponential transient. Conversely, if the auto-steering device has some k_1 and k_2 which give complex roots of characteristic equation, the response will be stable with some oscillations.

The auto-steering device should be able to select the proper control costants, k_1 and k_2 .

It is evident that the dynamically unstable ship can be made to be stable directionally with auto-steering control.

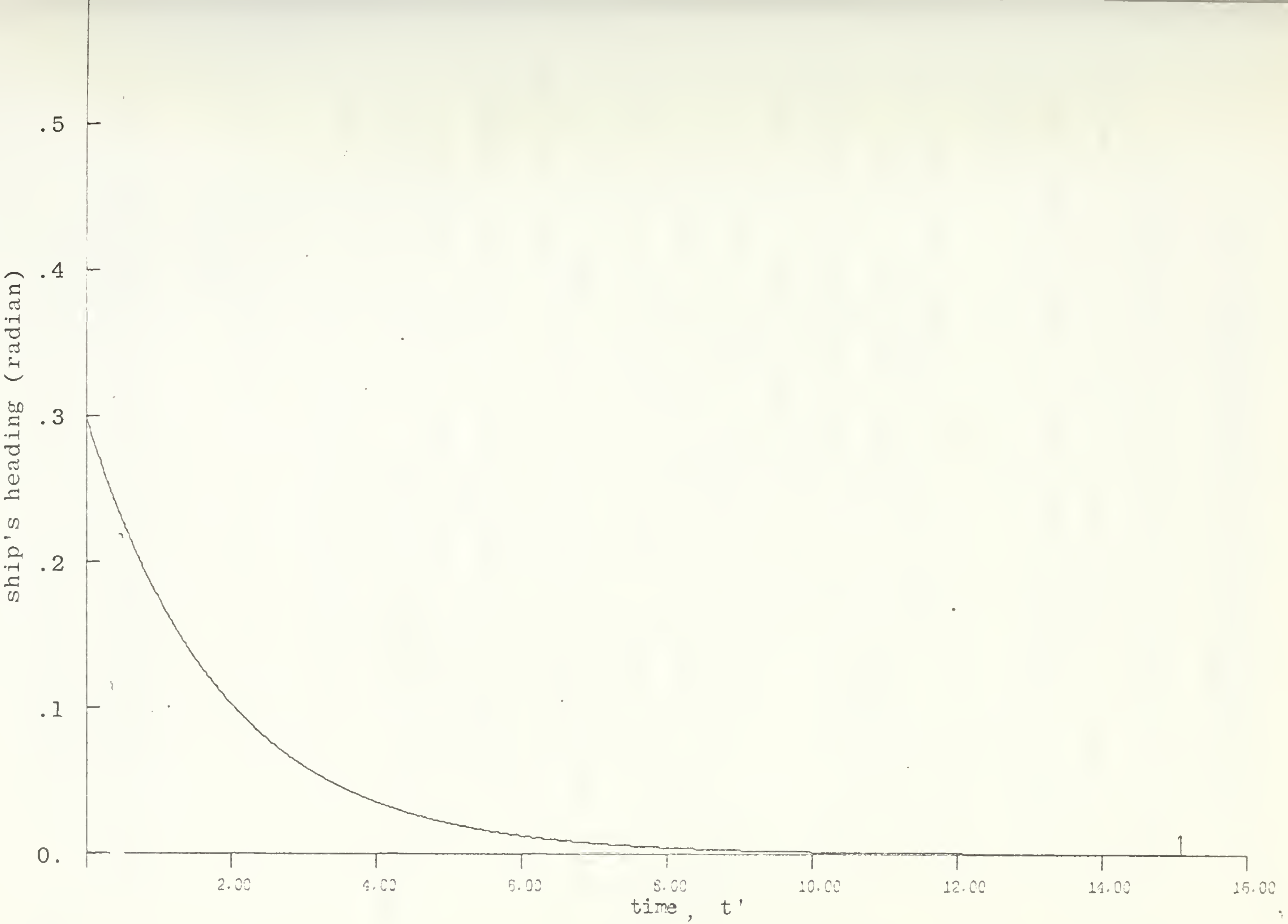


Figure 5-1. Response of ship's heading by auto-steering control after small disturbance.

IV. MANEUVERABILITY OF THE UNSTABLE SHIP

To predict the performance of a dynamically unstable ship as well as to get realistic predictions of maneuvers with large rudder angles, a non linear mathematical model is needed.

A. NON-LINEAR EQUATION OF SHIP MOTION.

In the non-linear analysis of the ship motion, the following assumptions which are rather typical in the literatures Refs. 2 and 8 were applied.

1) Terms of higher order than third order are neglected.

2) Rudder force and moment derivatives of higher than first order, and effects of rudder angular rate are negligible.

3) Odd terms in v, r, δ, $\dot{v}$ and $\dot{r}$ in the X-force equation and even terms in the same parameters of the Y-force and N-moment equations are eliminated due to symmetry.

4) Cross coupling terms between acceleration and velocity are neglected. Taking the Taylor Series expansion of Eq. (11) with the above assumptions:

$$\dot{u}' = \frac{f_1(u', v', r', \delta)}{m' - X'_{\dot{u}}}$$

$$\dot{v}' = \frac{(I_z' - N_{\dot{r}}') f_2(u', v', r', \delta) - (m'x_G' - Y_{\dot{r}}') f_3(u', v', r', \delta)}{(m' - Y_{\dot{v}}')(I_z' - N_{\dot{r}}') - (m'x_G' - N_{\dot{v}}')(m'x_G' - Y_{\dot{r}}')} \qquad (27)$$

$$\dot{r}' = \frac{(m'-Y'_{\dot{v}})f_3(u',v',r',\delta)-(m'x'_G-N'_{\dot{v}})f_2(u',v',r',\delta)}{(m'-Y'_{\dot{v}})(I'_z-N'_{\dot{r}})-(m'x'_G-N'_{\dot{v}})(m'x'_G-Y'_{\dot{r}})}$$

where

$$\begin{aligned}
f_1(u',v',r',\delta) = {} & X'_0+X'_u\Delta u'+\tfrac{1}{2}X'_{uu}\Delta u'^2+\tfrac{1}{6}X'_{uuu}\Delta u'^3+\\
& \tfrac{1}{2}X'_{vv}v'^2+(\tfrac{1}{2}X'_{rr}+m'x'_G)r'^2+\tfrac{1}{2}X'_{\delta\delta}\delta^2+\\
& \tfrac{1}{2}X'_{vvu}v'^2\Delta u'+\tfrac{1}{2}X'_{rru}r'^2\Delta u'+\tfrac{1}{2}X'_{\delta\delta u}\delta^2\Delta u'+\\
& (X'_{vr}+m')v'r'+X'_{v\delta}v'\delta+X'_{r\delta}r'\delta+X'_{vru}v'r'\Delta u'+\\
& X'_{v\delta u}v'\delta\Delta u'+X'_{r\delta u}r'\delta\Delta u'\\
f_2(u',v',r',\delta) = {} & Y'_0+Y'_{0u}\Delta u'+Y'_{0uu}\Delta u'^2+Y'_v v'+\tfrac{1}{6}Y'_{vvv}v'^3+\\
& \tfrac{1}{2}Y'_{vrr}v'r'^2+\tfrac{1}{2}Y'_{v\delta\delta}v'\delta^2+Y'_{vu}v'\Delta u'+\tfrac{1}{2}Y'_{vuu}v'\Delta u'^2+\\
& Y'_\delta\delta+\tfrac{1}{6}Y'_{\delta\delta\delta}\delta^3+\tfrac{1}{2}Y'_{\delta vv}\delta v'^2+\tfrac{1}{2}Y'_{\delta rr}\delta r'^2+\\
& Y'_{\delta u}\delta\Delta u'+\tfrac{1}{2}Y'_{\delta uu}\delta\Delta u'^2+Y'_{vr\delta}v'r'\delta\\
f_3(u',v',r',\delta) = {} & N'_0+N'_{0u}\Delta u'+N'_{0uu}\Delta u'^2+N'_v v'+\tfrac{1}{6}N'_{vvv}v'^3+\\
& \tfrac{1}{2}N'_{vrr}v'r'^2+\tfrac{1}{2}N'_{v\delta\delta}v'\delta^2+N'_{vu}v'\Delta u'+\tfrac{1}{2}N'_{vuu}v'\Delta u'^2+\\
& (N'_r-m'x'_G u')r'+\tfrac{1}{6}N'_{rrr}r'^3+\tfrac{1}{2}N'_{rvv}r'v'^2+\tfrac{1}{2}N'_{r\delta\delta}r'\delta^2+\\
& N'_{ru}r'\Delta u'+\tfrac{1}{2}N'_{ruu}r'\Delta u'^2+N'_\delta\delta+\tfrac{1}{6}N'_{\delta\delta\delta}\delta^3+\\
& \tfrac{1}{2}N'_{\delta vv}\delta v'^2+\tfrac{1}{2}N'_{\delta rr}\delta r'^2+N'_{\delta u}\delta\Delta u'+\\
& \tfrac{1}{2}N'_{\delta uu}\delta\Delta u'^2+N'_{vr\delta}v'r'\delta
\end{aligned} \tag{28}$$

Integrating Eq. (27)

$$
\begin{aligned}
u'(t') &= \int_0^{t'} \dot{u}'(t')\,dt' \\
v'(t') &= \int_0^{t'} \dot{v}'(t')\,dt' \\
r'(t') &= \int_0^{t'} \dot{r}'(t')\,dt' \\
\psi(t') &= \int_0^{t'} r'(t')\,dt'
\end{aligned}
\qquad (29)
$$

Note: The time domain should be non-dimensional.

So far the reference axes x, y and z are fixed to the moving ship. But it is convenient to take the reference axes x_o and y_o fixed to the earth in order to determine the path of the ship. Adopting the figure from Ref. 1 gives

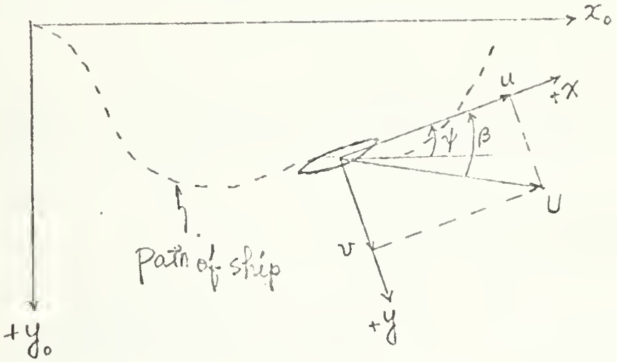


FIGURE 6. Relations between axes fixed in the earth and axes fixed in the ship.

The following relationships may be written as

$$\begin{aligned}\dot{x}_o(t) &= u(t)\cos\psi(t) - v(t)\sin\psi(t)\\ \dot{y}_o(t) &= u(t)\sin\psi(t) + v(t)\cos\psi(t)\end{aligned} \qquad (30)$$

$$\begin{aligned}x_o(t) &= \int_0^t \dot{x}_o(t)\,dt\\ y_o(t) &= \int_0^t \dot{y}_o(t)\,dt\end{aligned} \qquad (31)$$

Non-dimensionalizing Eqs. (30) and (31) using the definitions,

$$x' = \frac{x_o}{L}\ ,\quad \dot{x}' = \frac{\dot{x}_o}{U}\ ,\quad y' = \frac{y_o}{L}\ ,\quad \dot{y}' = \frac{\dot{y}_o}{U}\ ,\quad t' = t\,\frac{U}{L}$$

and,

$$\frac{x_o}{L} = \int \frac{\dot{x}_o}{L}\,dt = \int \frac{\dot{x}_o}{U}\,\frac{U}{L}\,dt$$

or

$$x'(t') = \int \dot{x}'\,dt' \qquad (32)$$

Likewise,

$$y'(t') = \int \dot{y}'\,dt'$$

B. TURNING CIRCLE WITH SELECTED RUDDER DEFLECTION

Fig. 7 shows the typical rudder deflection with respect to time in a turning maneuver.

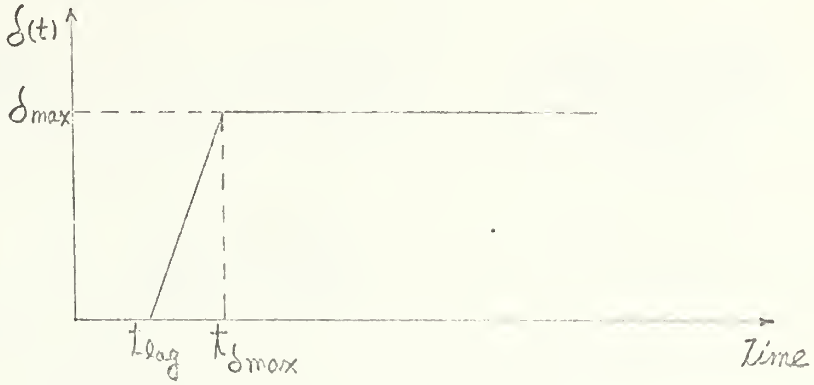


FIGURE 7. Rudder deflection vs. time in turning maneuver

Using the ramp function for rudder deflection, then

$$\delta(t) = \alpha\left[ramp(t-t_{lag}) - ramp(t-t_{\delta max})\right] \quad (33)$$

where

δ = rudder deflection
ramp (t) = ramp function
α = rudder deflection rate
t_{lag} = time delay
$t_{\delta max}$ = time at δ_{max}.

Calculation of steady turning radius.

The relation of the angular velocity with linear velocity gives:

(radius) x (angular velocity) = (linear velocity)

or $$\text{Radius} = \frac{U(t)}{r(t)} = \frac{\sqrt{u^2(t)+v^2(t)}}{r(t)}$$

Taking non-dimensional terms

$$\frac{Radius}{L} = \frac{U}{UL}\,\frac{\sqrt{u^2+v^2}}{r} = \frac{\sqrt{(u/U)^2+(v/U)^2}}{r\left(\frac{U}{L}\right)}$$

or $$Rad' = \frac{\sqrt{(u')^2+(v')^2}}{L} \qquad (34)$$

where $$Rad' = \frac{Radius}{L}$$

Figure 8 shows the typical turning trajectory and terminologies corresponding to Table 5. Eqs. (31) and (34) were simulated for the 5 models using computer program 5 in turning circle maneuvers with a rudder deflection rate of 2 degrees per second for the prototype (L = 500') with zero time lag and constant final angles of 10°, 20° and 35°. The results are presented in Table 5 and Figs. 8a, 8b and 8c.

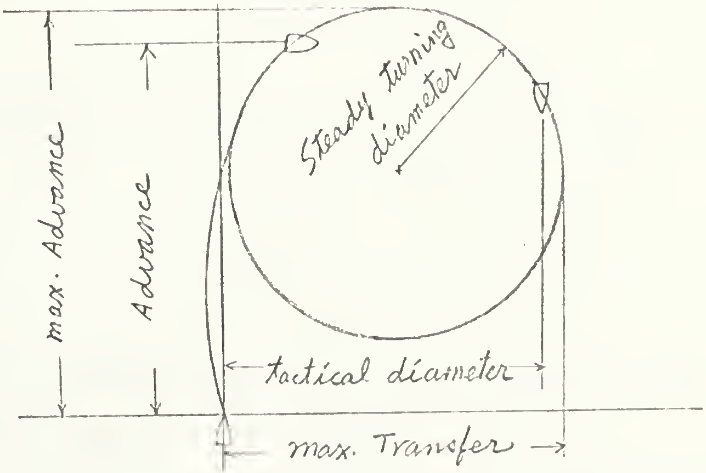


FIGURE 8. Definition of turning circle parameters.

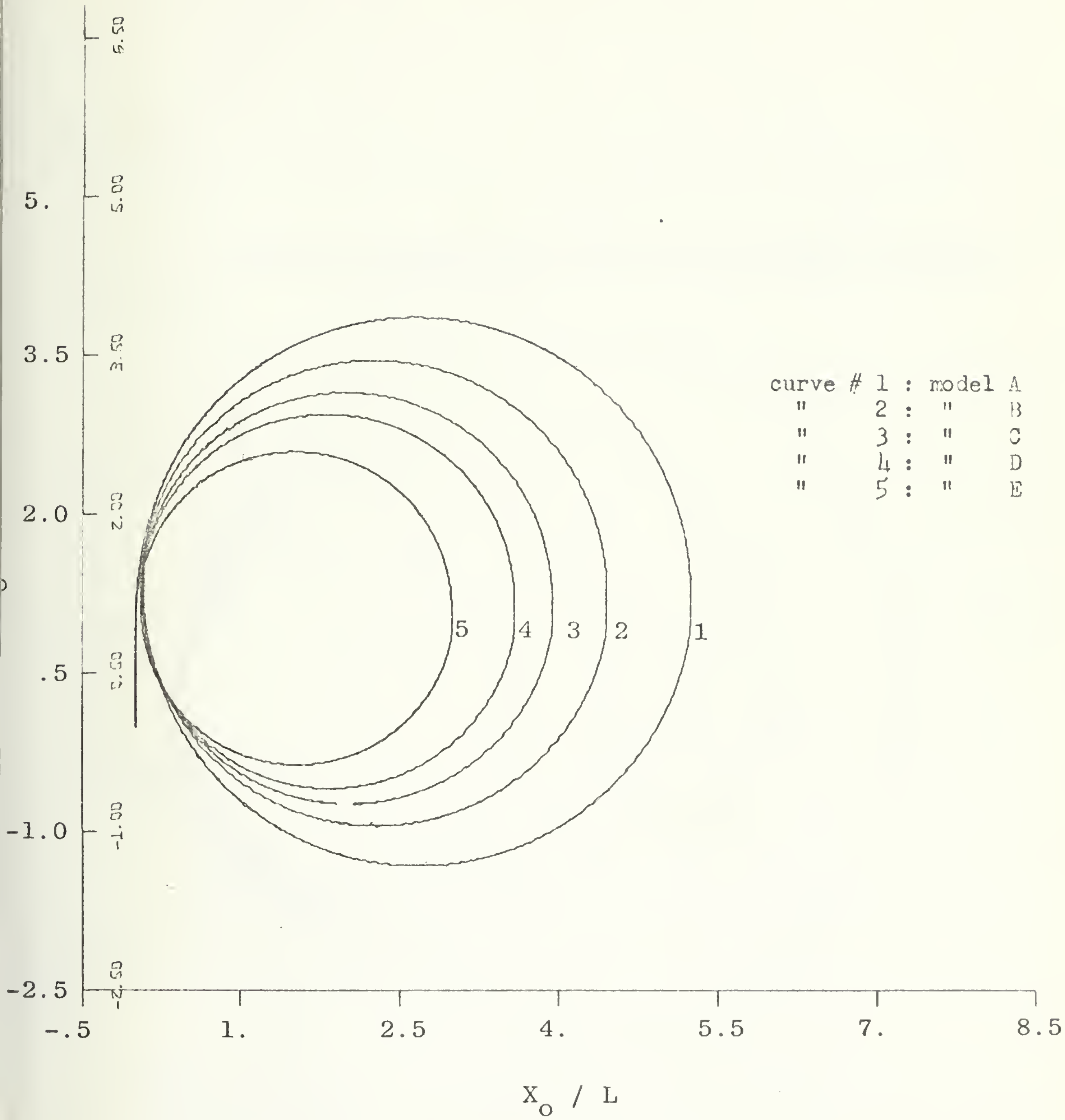


FIGURE 8a. Turning circle of five models with 35 degrees of rudder angle.

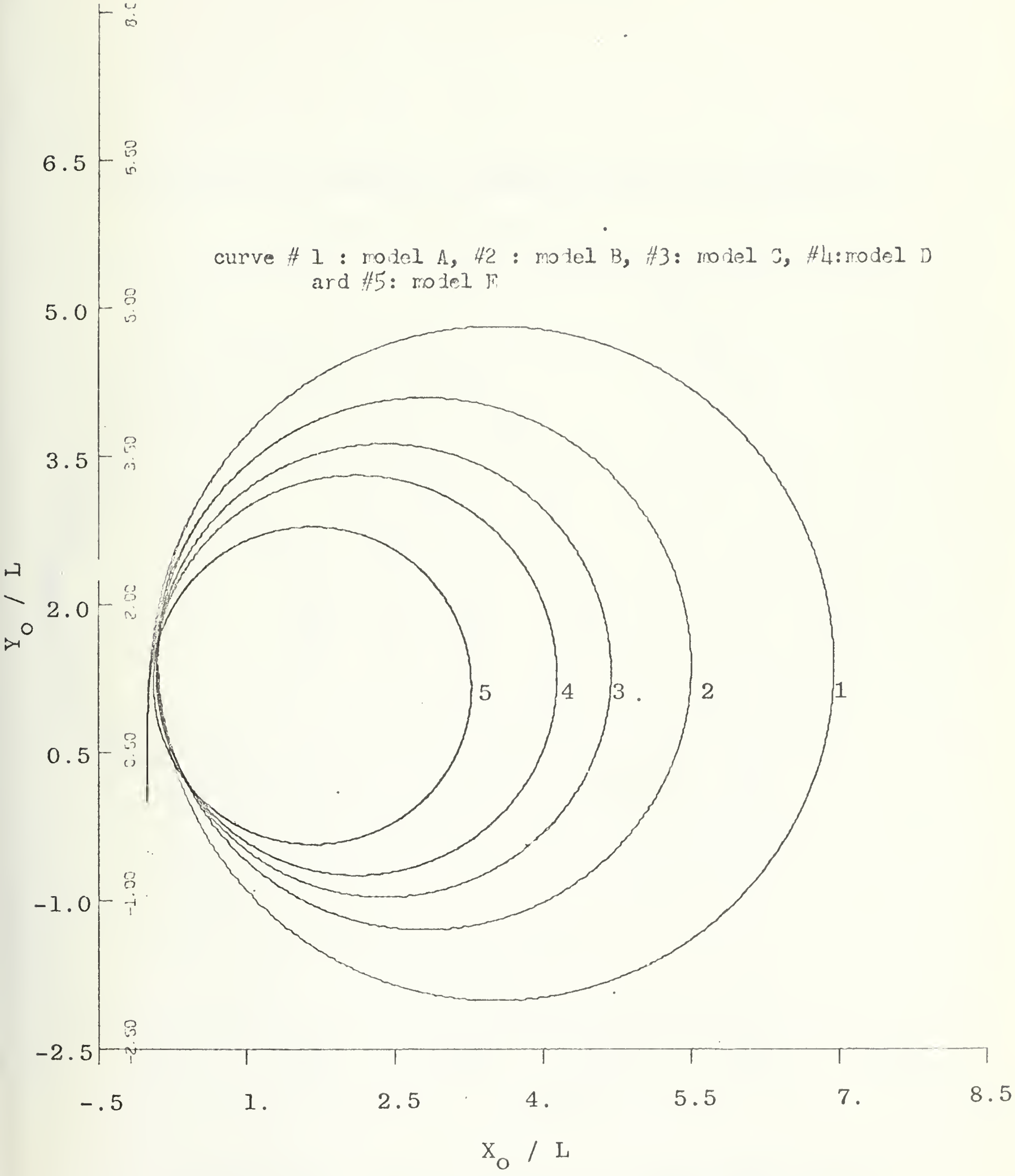


FIGURE 8b. Turning circle of five models with 25 degrees of rudder angle.

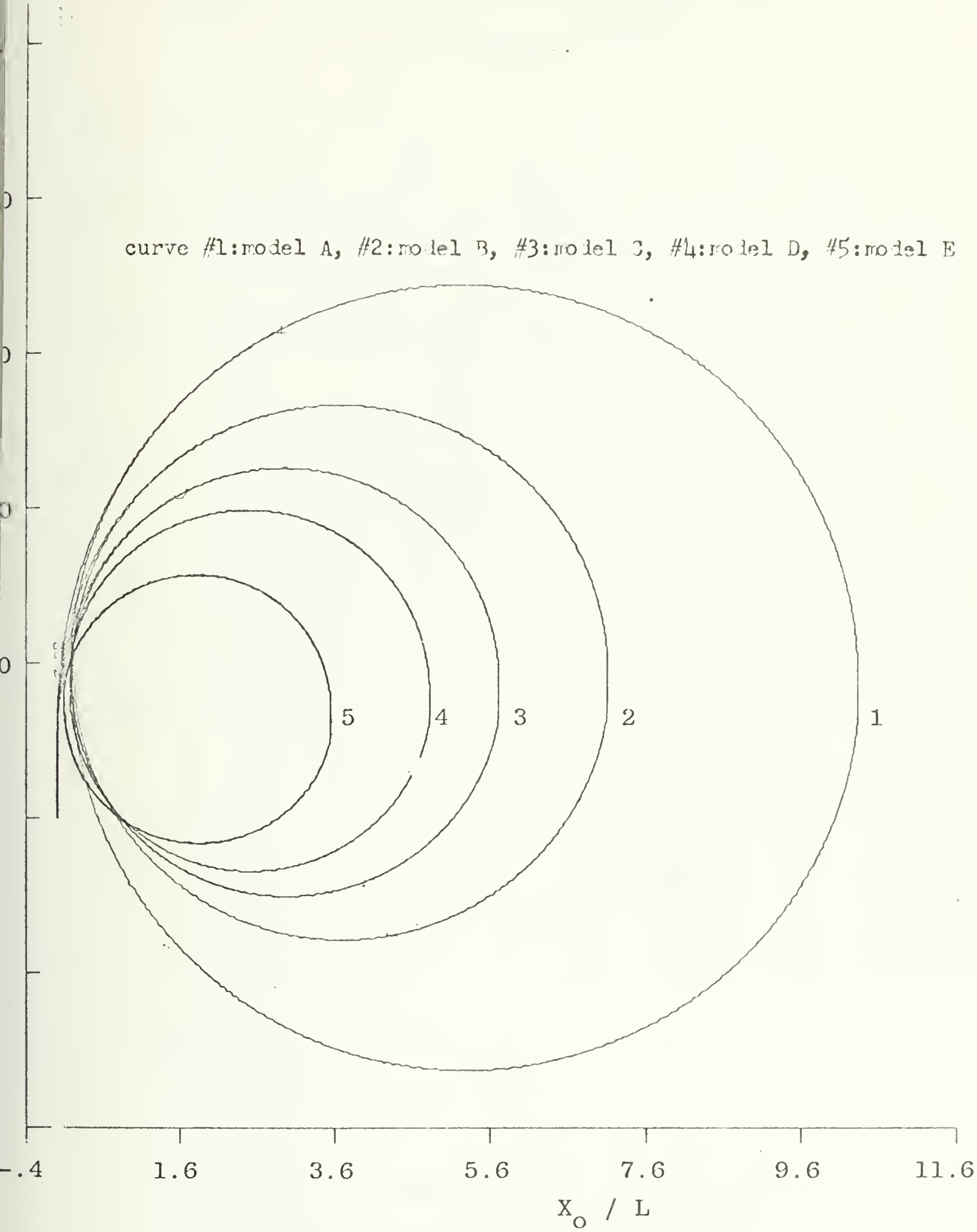


FIGURE 8c. Turning circle of five models with 10 degrees of rudder angle.

TABLE V. CHARACTERISTICS OF MODEL A,B,C,D AND E IN TURNING CIRCLE

δ	Model	Adv.	Max. Adv.	Tran.	T.D.	Max. Tran.	S.T.R.	t'/rev
10°	A	6.86	6.8825	4.86	10.30	10.322	5.0722	31.6
	B	5.30	5.3224	3.25	7.08	7.1079	3.4498	21.4
	C	4.49	4.5088	2.65	5.88	5.7079	2.7611	17.2
	D	3.94	3.9667	2.18	4.79	4.8122	2.3290	14.6
	E	3.08	3.1277	1.45	3.51	3.5490	1.7295	10.8
20°	A	4.80	4.8269	3.16	6.91	6.9411	3.4177	21.2
	B	4.07	4.0996	2.40	5.46	5.4962	2.6946	16.8
	C	3.60	3.6306	2.06	4.66	4.6863	2.2951	14.2
	D	3.29	3.3134	1.80	4.10	4.1379	2.0266	12.6
	E	2.70	2.7892	1.35	3.23	3.2719	1.6070	10.0
35°	A	3.840	3.8685	2.33	5.2203	5.2433	2.6364	16.2
	B	3.430	3.4584	1.98	4.435	4.4551	2.1938	13.6
	C	3.120	3.1538	1.77	3.900	3.9380	1.9385	12.0
	D	2.918	2.9475	1.49	3.540	3.5804	1.760	10.8
	E	2.550	2.5943	1.15	2.790	2.9890	1.4727	8.6

Note: Adv. = Advance/L

Tran. = Transfer/L

T.D. = Tactical Diameter/L

S.T.R. = Steady turn radius/L

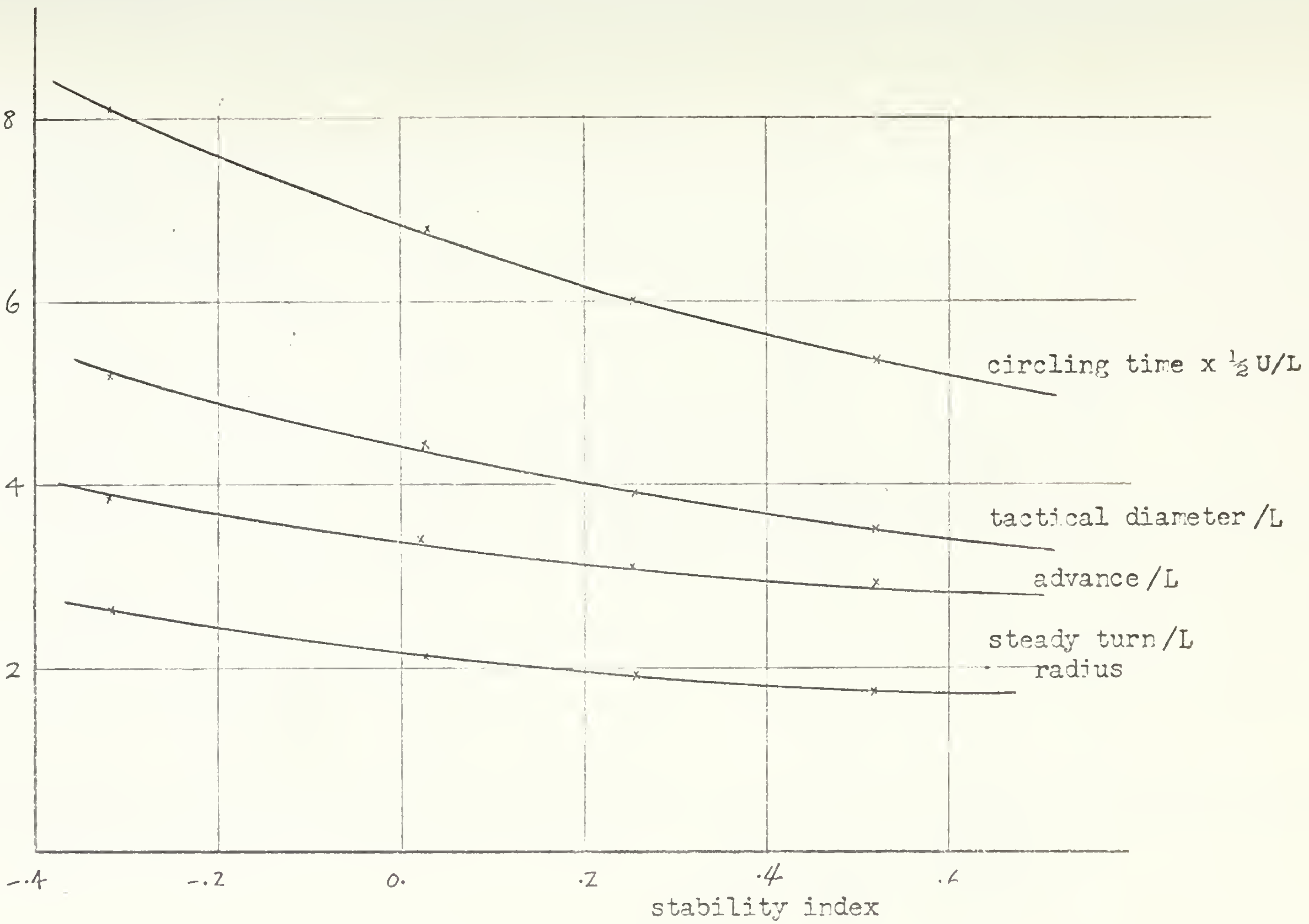


Fig.9 Relations between turning circle parameters and stability index at 35 degree of rudder deflection.

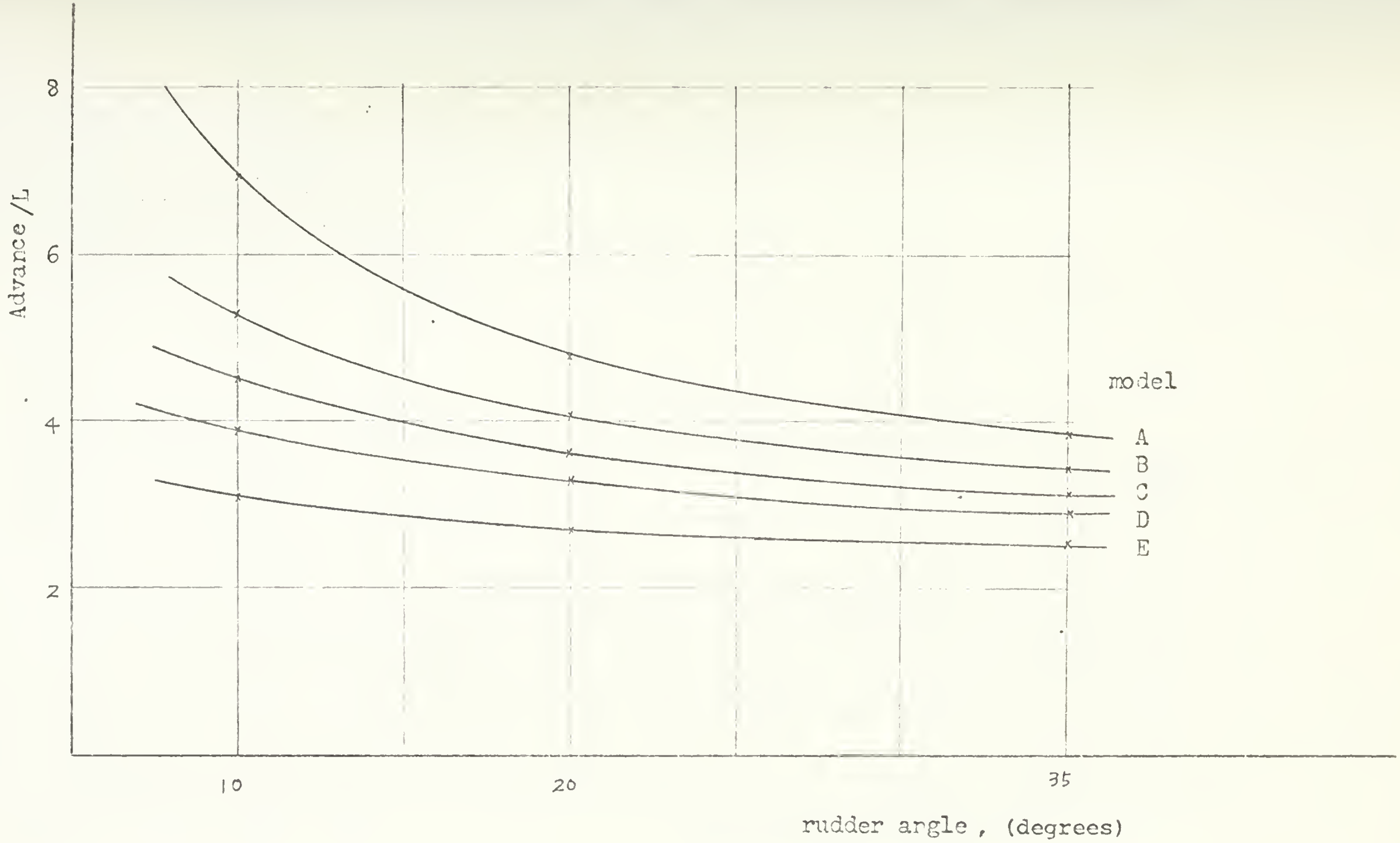


Fig.10 Advance vs. rudder angle in turning circle.

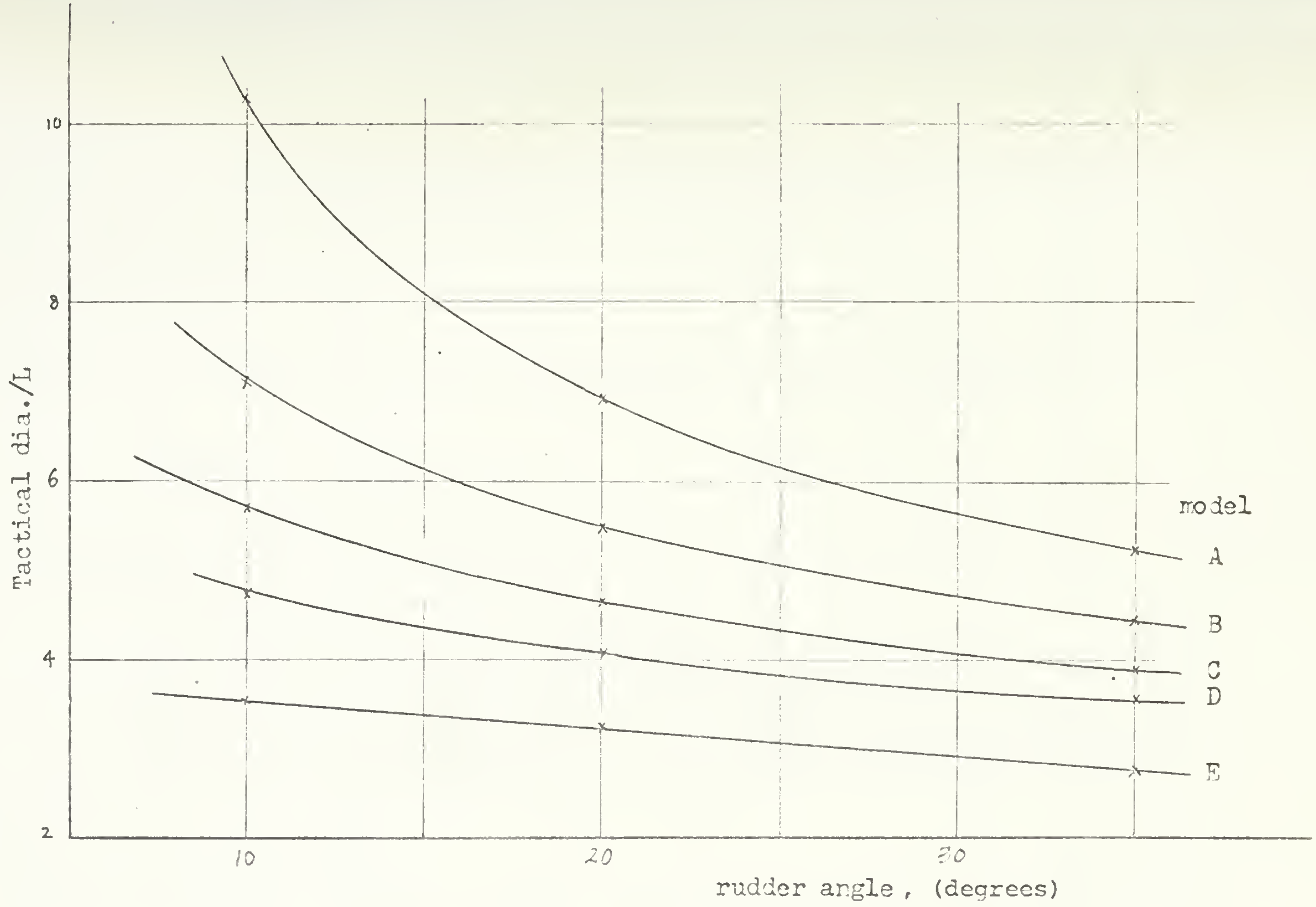


Fig.11 Tactical diameter vs. Rudder angle in turning circle.

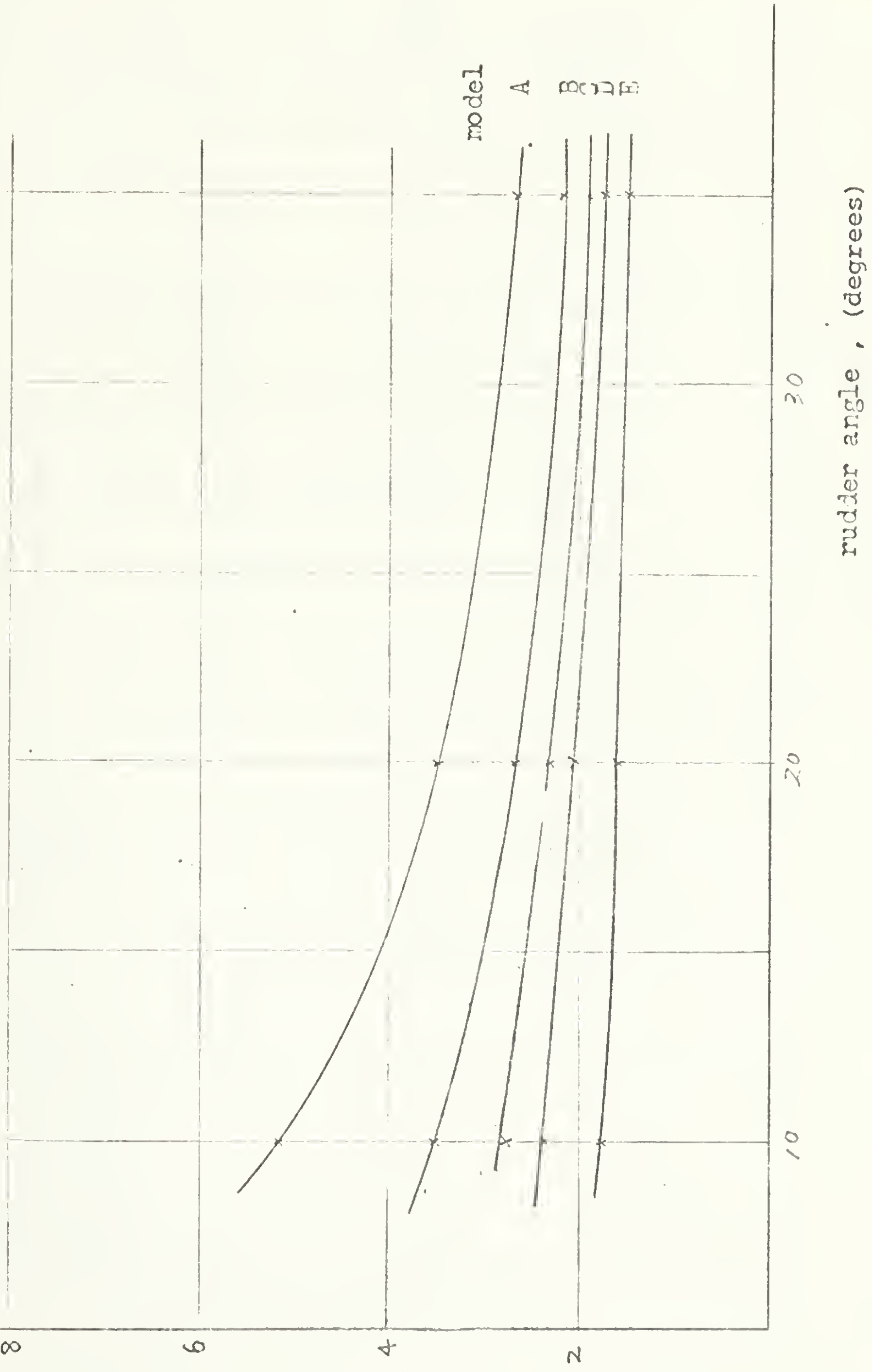


Fig. 12 Steady turning radius vs. rudder angle.

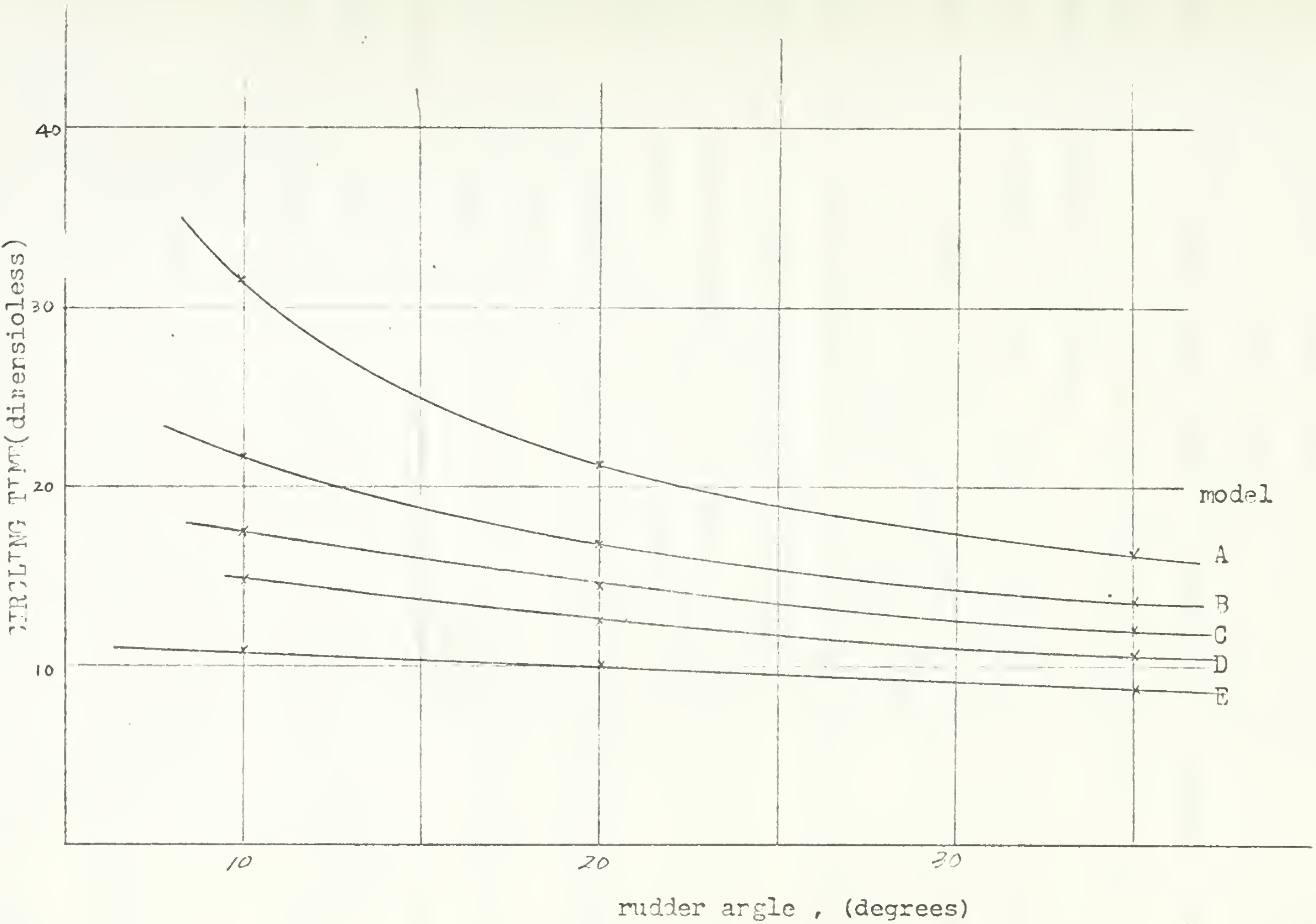


Fig.13 Circling time vs. rudder angle in turning circle.

Discussion of turning circle maneuver

1) As shown in Fig. 9, the dynamically less stable ship has a smaller tactical diameter, advance, steady turning radius and circling time than the more stable ship, because the yawing rate of the unstable ship is greater than that of stable ship at the same rudder deflection. However, the turning circle parameters are becoming constant as the stability index increases.

2) As shown in Figs. 10, 11, 12 and 13, the rate of change of the turning circle parameters versus the rudder deflection is high at the small rudder angle. In other words, the above parameters are less sensitive to rudder angle as the rudder deflection increases.

3) The more unstable the ship, the less sensitive the above parameters are to the rudder deflection.

1. The Effect of the Location of the Rudder on the Turning Circle

A change in the location of the rudder has multiple effects on the ship motion. One is it's effect on dynamical stability, which is the same as the effect of a movable fin described in section (II-C-1) and the other is it's effect on maneuverability.

As described in Appendix A and Ref. 1,

1) Y_v, $Y_{\dot{v}}$ and Y_δ are independent of the position of the rudder.

2) N_v, Y_r, $N_{\dot{v}}$ and $Y_{\dot{r}}$ are less positive if the rudder moves forward from the stern.

3) N_r and $N_{\dot{r}}$ are less negative if the rudder moves from the stern to the midship position and less positive from midship to the bow.

4) Since the relative flow passing the ship depends on the position considered, the attack angle depends on the location of the rudder. Forward of the pivot point the attack angle is larger than rudder angle, at the pivot point the attack angle is equal to rudder angle and aft of the pivot point the attack angle is smaller than the rudder angle.

5) N_δ strongly depends on the location of the rudder since $N_\delta = Y_\delta \cdot x_f$ where x_f is the distance of the rudder position from midship and is negative aft of midship. N_δ is less negative if the rudder moves forward to the bow, but $N_\delta \delta$ is less positive if δ is negative. This less positive effect decreases the yawing rate, r, due to decreases of f_3 in Eqs. (23) and (28). Decreasing r in turn causes the turning radius to increases.

On the other hand, the movable fin effect of shifting the rudder forward makes the ship more unstable which makes the turning radius decrease. These two compromising effects are shown in Fig. 14 and 15 which were obtained from computer program 6. In computer program 6, model C was simulated for the evaluation of the effect of the rudder position on turning maneuvers.

As shown on Fig. 14 which represents the effect of the rudder position on the turning radius without replacement of

TABLE VI. THE EFFECT OF RUDDER POSITION ON THE TURNING CIRCLE OF MODEL C WITHOUT A FIN AT THE STERN.

x_f'	t'/rev.	v'	r'	Rad$'$	σ_1
-0.5	7.12	-0.1617	0.4413	2.2960	0.25616
-0.4	6.84	-0.16571	0.45764	2.25	
-0.3	6.72	-0.1701	0.4688	2.164	0.5482
-0.22	6.64	-0.1735	0.4734	2.144	0.6492
-0.2	6.6	-0.1743	0.4739	2.14	
-0.17	6.64	-0.17556	0.4742	2.1412	0.70581
-0.10	6.64	-0.1784	0.424	2.15	
0	6.76	-0.1822	0.4639	2.19	0.8552
+0.1	7.0	-0.1856	0.4485	2.268	0.90969
0.15	7.16	-0.1871	0.4383	2.32	
0.20	7.44	-0.1885	0.4265	2.386	0.9394
0.21	7.43	-0.18869	0.42398	2.4	0.9410
0.22	7.45	-0.1889	0.4214	2.415	0.9424
0.24	6.08	+0.1345	-0.51396	1.963	0.9445
0.25	6.08	+0.1352	-0.5163	1.955	
0.27	6.04	+0.1366	-0.5205	1.94	
0.30	6.0	+0.1385	-0.5260	1.92	0.9456
0.34	5.92	0.14099	-0.53204	1.898	0.9420
0.5	5.8	0.1493	-0.5425	1.804	0.8980

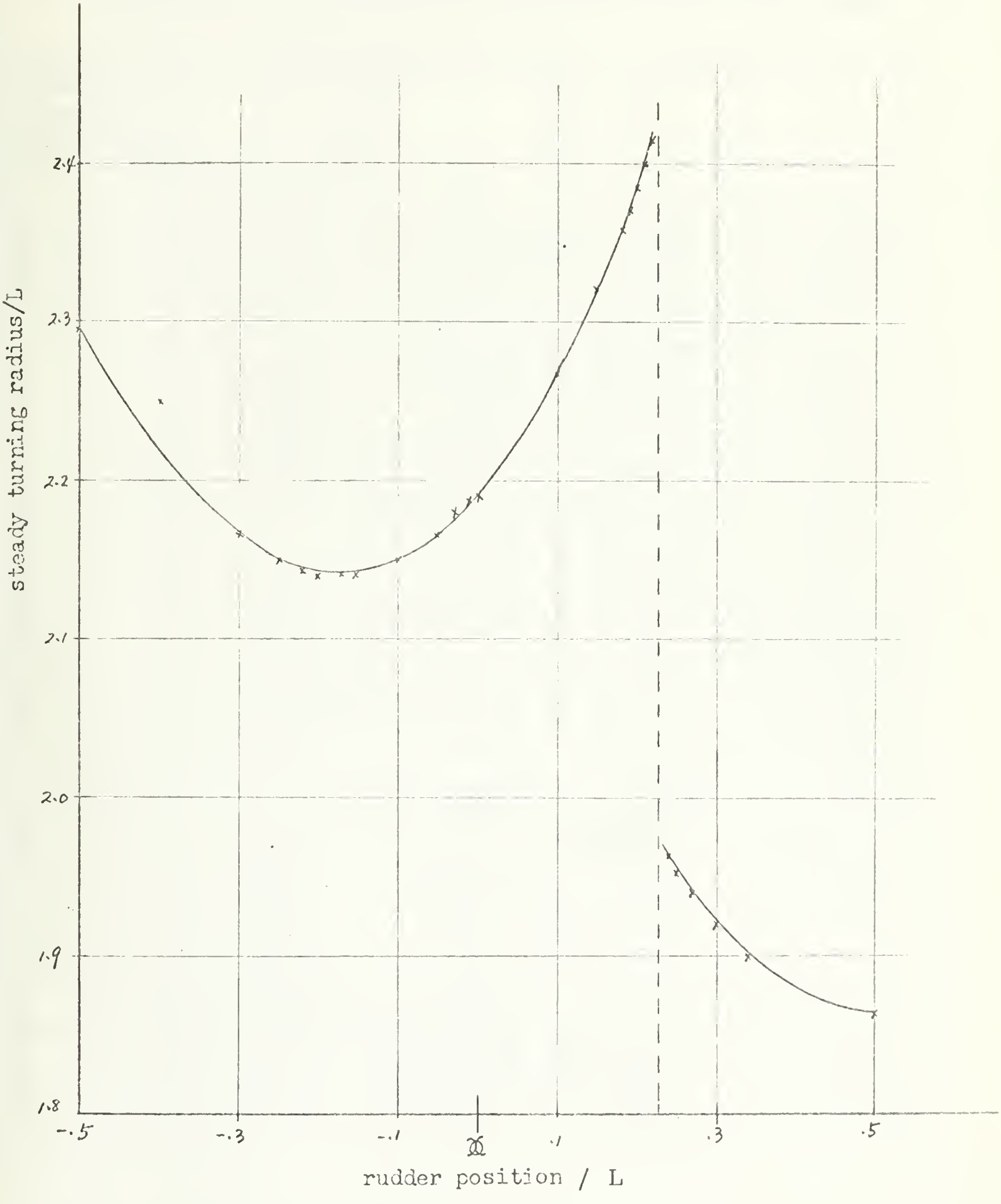


Fig.14 The effect of the position of the rudder on turning radius without a fin at the stern.

TABLE VIa. THE EFFECT OF RUDDER POSITION ON THE TURNING CIRCLE WITH A STABILIZING FIN AT THE STERN.

(Rudder Position from ℄)/L	v'	r'	Rad'	σ_1
-0.5	-0.1193	0.3053	3.299	-0.3804
-0.4	-0.1233	0.3112	3.238	-0.2570
-0.35	-0.1252	0.3125	3.225	-0.1937
-0.33	-0.1259	0.3127	3.223	-0.1683
-0.3	-0.1269	0.3126	3.225	-0.1305
-0.25	-0.1286	0.3113	3.238	-0.0625
-0.2	-0.13	0.3086	3.268	-0.0088
-0.1	-0.1322	0.2983	3.382	+0.1001
0.	-0.1333	0.2810	3.590	+0.1892
0.2	-0.1292	0.2220	4.542	+0.2948
0.24	-0.1267	0.2054	4.908	+0.3039
0.26	-0.1251	0.1961	5.138	+0.3070
0.27	-0.1242	0.1913	5.268	+0.308
0.28	+0.0572	-0.2827	3.544	+0.309
0.30	+0.0621	-0.2971	3.373	+0.310
0.4	+0.0779	-0.3413	2.939	+0.305
0.5	+0.0874	-0.3639	2.758	+0.2820
	-0.1617	0.4413	2.296	+0.2561

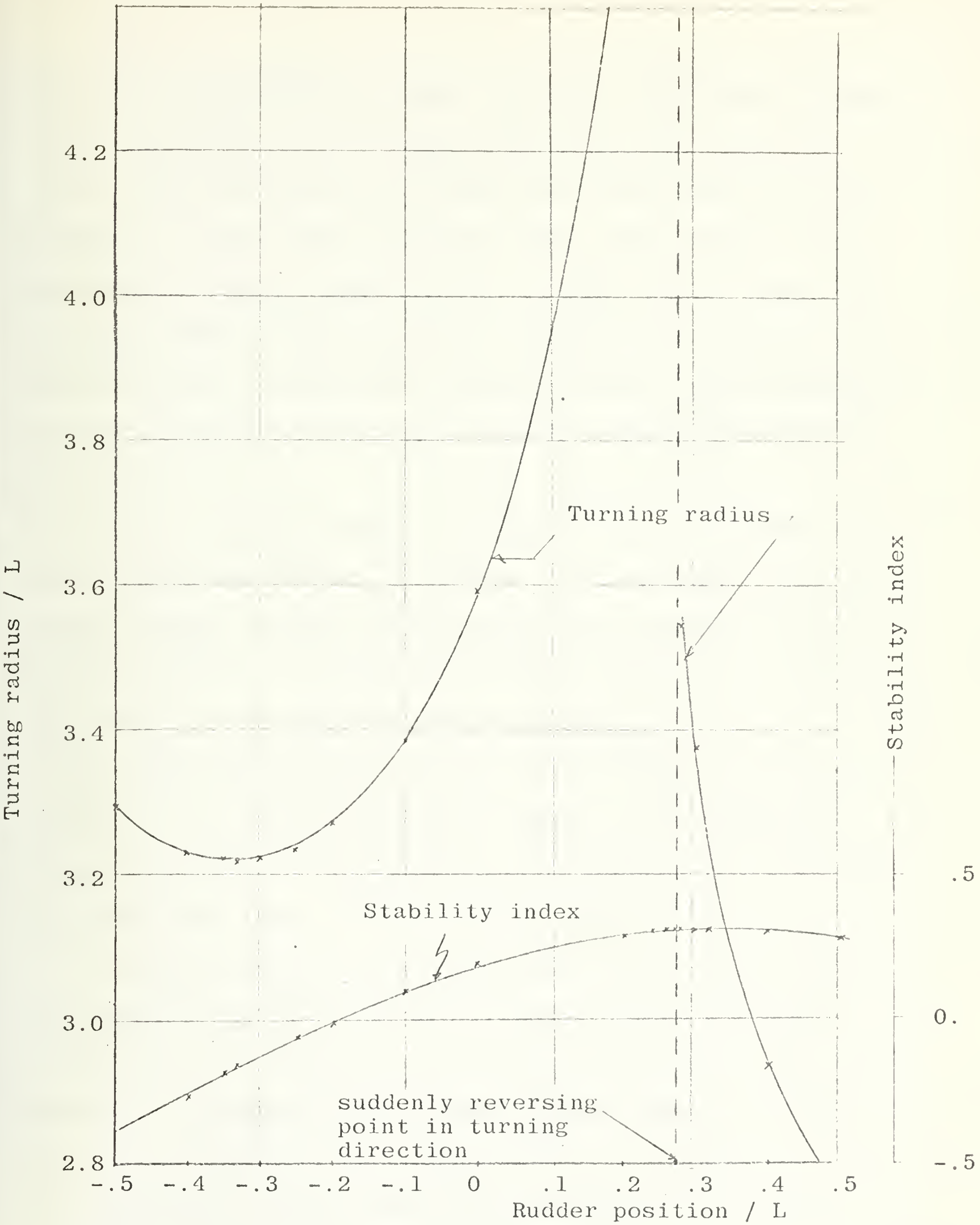


FIGURE 15. The effect of the rudder position on turning circle of model C with a stabilizing fin at the stern.

the fin at the original rudder location, the steady turning radius decreased as the rudder was moved from -0.5L to -0.2L, and increased as the rudder was moved from -0.2L to +0.22L. At the rudder position 0.24L, the turning radius decreased suddenly from 2.415L to 1.963L. Also, since the turning moment at this point was reversed in direction, the ship suddenly swings to the opposite side. This phenomenon is the characteristic of the unstable ship as described in section IV-D.

If the turning moment is decreased due to shifting the position of the rudder, the yawing rate decreases until the rudder change occurs. After changing the turning direction, the magnitude of yaw rate was suddenly increased. Finally, the turning radius was decreased markedly.

To find the optimum position of the rudder for proper stability and maneuverability of model C, the rudder was moved forward to the bow and in it's location a stabilizing fin was installed.

As shown in Fig. 15 and Table VIa, the turning radius at rudder position -0.33L is a minimum before a sudden change of rotation and in addition the stability index is negative. Although this minimum turning radius (3.223L) is greater than that of the original model C (2.296L), it is an acceptable optimum rudder position due to it's better stability index, i.e., negative index.

C. ZIG-ZAG MANEUVER

According to the standard zig-zag maneuver, the relation of the rudder deflection (δ) and ship's heading (ψ) is as described in Fig. 16.

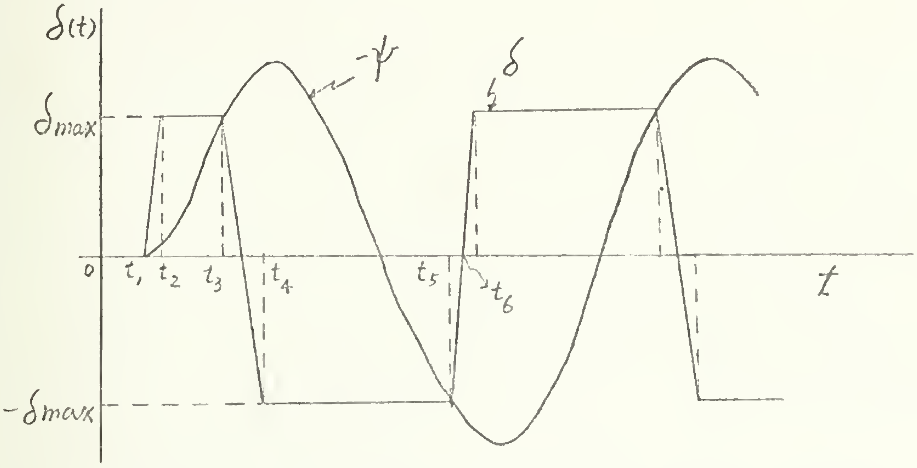


FIGURE 16. Relations between rudder deflection, ship's heading and time in standard zig-zag maneuver.

Using a ramp function for the rudder displacement, the rudder angle is given by:

$$\delta(t) = \alpha[ramp(t-t_1) - ramp(t-t_2) - ramp(t-t_3) + ramp(t-t_4) + ramp(t-t_5)]$$

for 1 cycle until $t \leq t_6$,

where

α = rudder rate (degree/sec.)
$ramp(t)$ = ramp function
t_1 = time delay of rudder
t_2 = time at $\delta = \delta_{max}$
t_3 = time at $\delta = -\psi$
t_4 = time at $\delta = -\delta_{max}$
t_5 = time at $\delta = \psi$

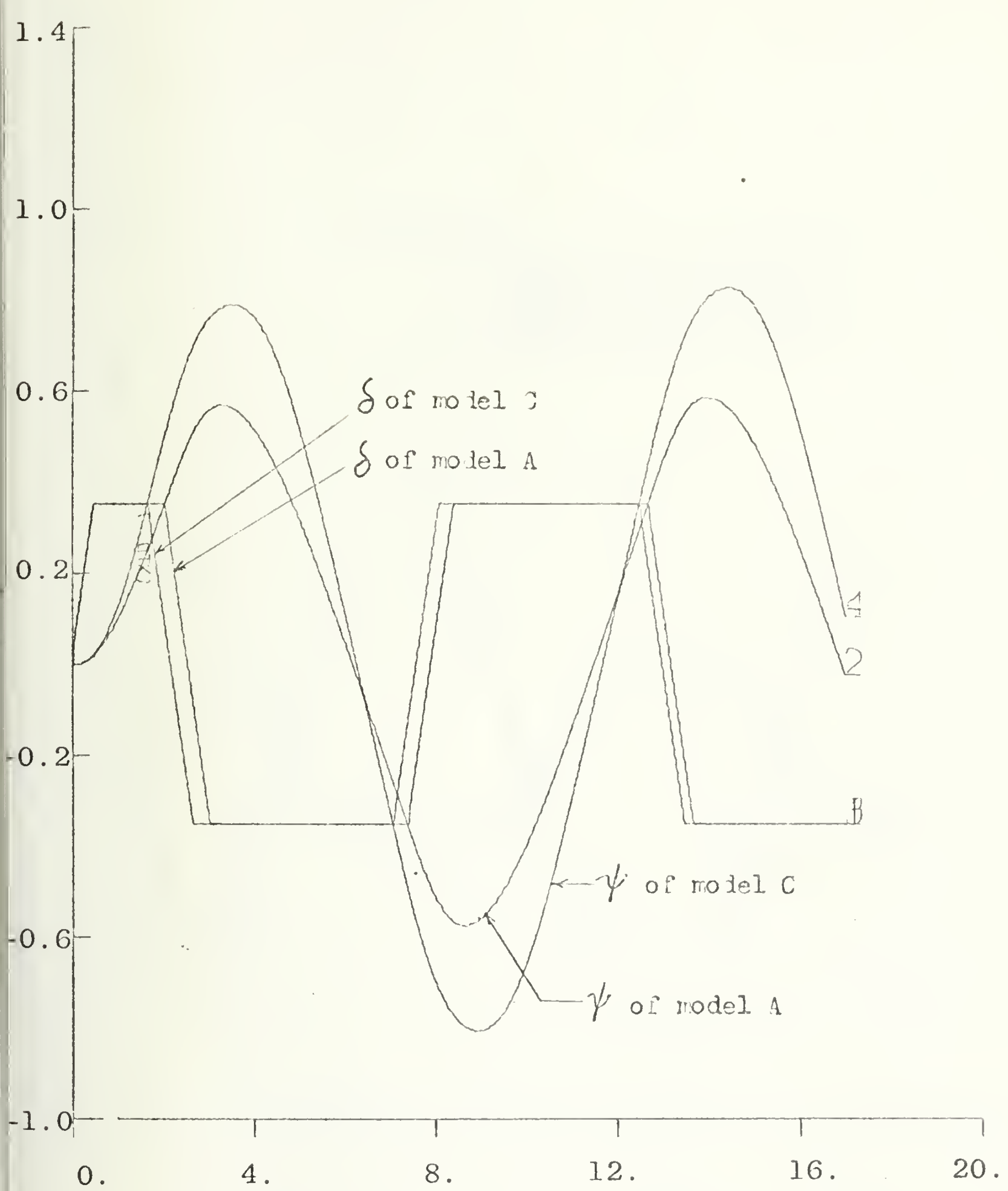


FIGURE 17. Rudder Deflection and Ship's Heading in Z-maneuver.

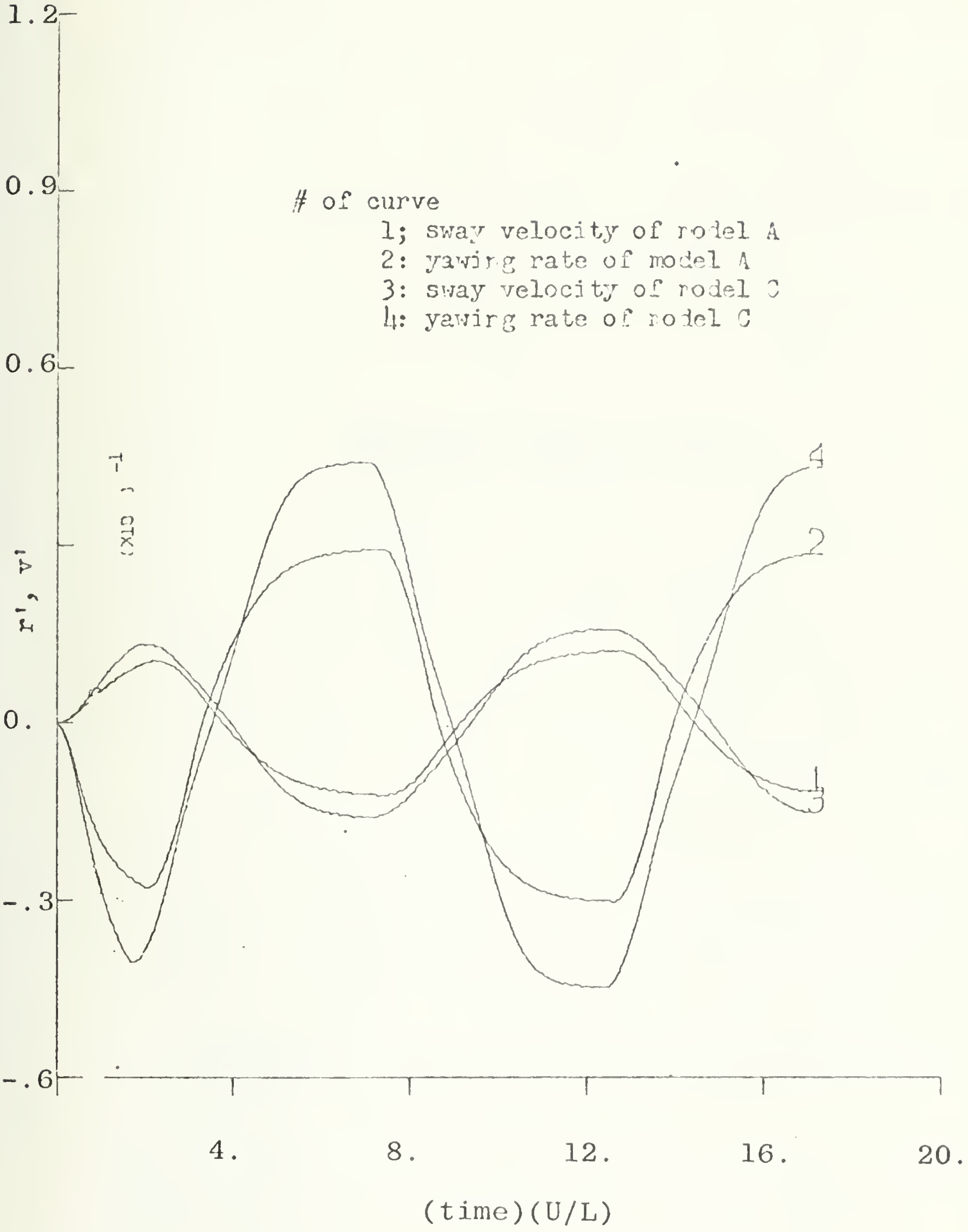


FIGURE 18. Yawing Rate and Sway Velocity in Z-maneuver.

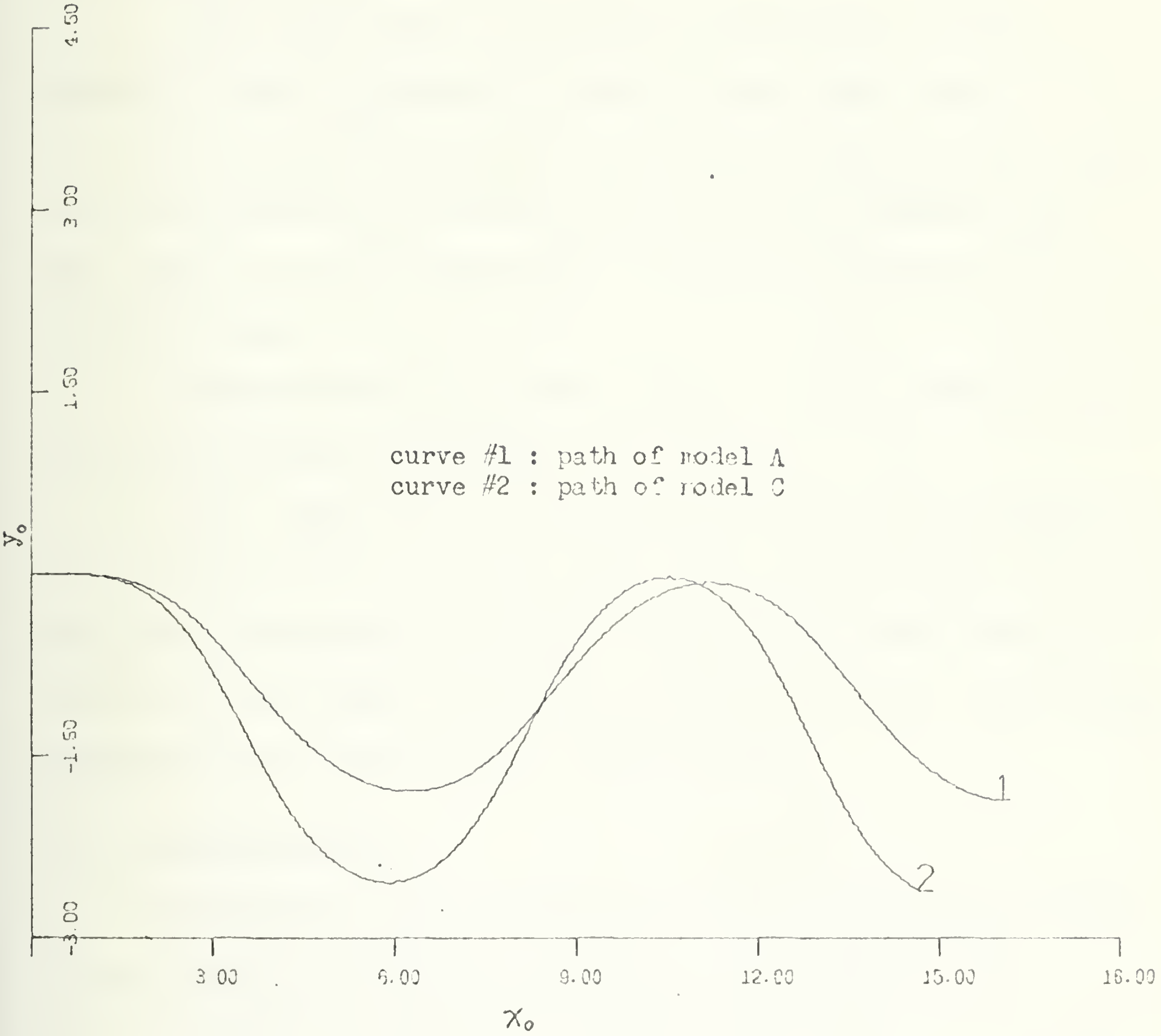


FIGURE 19. Path Trajectory in Z-maneuver.

In computer program 8, the standard zig-zag maneuvers were simulated for model A and C. As shown in Figs. 18, 19 and 20 which represent the results of the program 8, the following can be observed:

1) The amplitude of overshoot in yaw angle of the unstable ship is larger than that of the stable ship.

2) Although the first reversing execution of the rudder of the unstable ship is earlier than for the stable ship, the period of the stable ship is shorter than that of the unstable ship.

3) Amplitude of overshoot of path in the unstable ship is greater than stable ship.

4) The reason for 1), 2) and 3) is that the yaw rate of the unstalbe ship is greater than that of the stable ship but the sway velocities are almost the same for either ship so that it takes longer for the unstable ship to get the reverse heading after reversing the rudder angle.

D. SPIRAL MANEUVER

The standard spiral maneuver of model C was carried out using DSL/360 in computer program 9. The process was started with 10 degrees port rudder and 1 degree of deflecting increment and ended with 10 degrees starboard rudder. The process was repeated.

Figure 20 and Table 7 show the results of computer program 9.

During the spiral test of model C, the angular velocity was reduced suddenly from $r' = 0.19$ to $r' = 0.28$ at $\delta = 1.5$

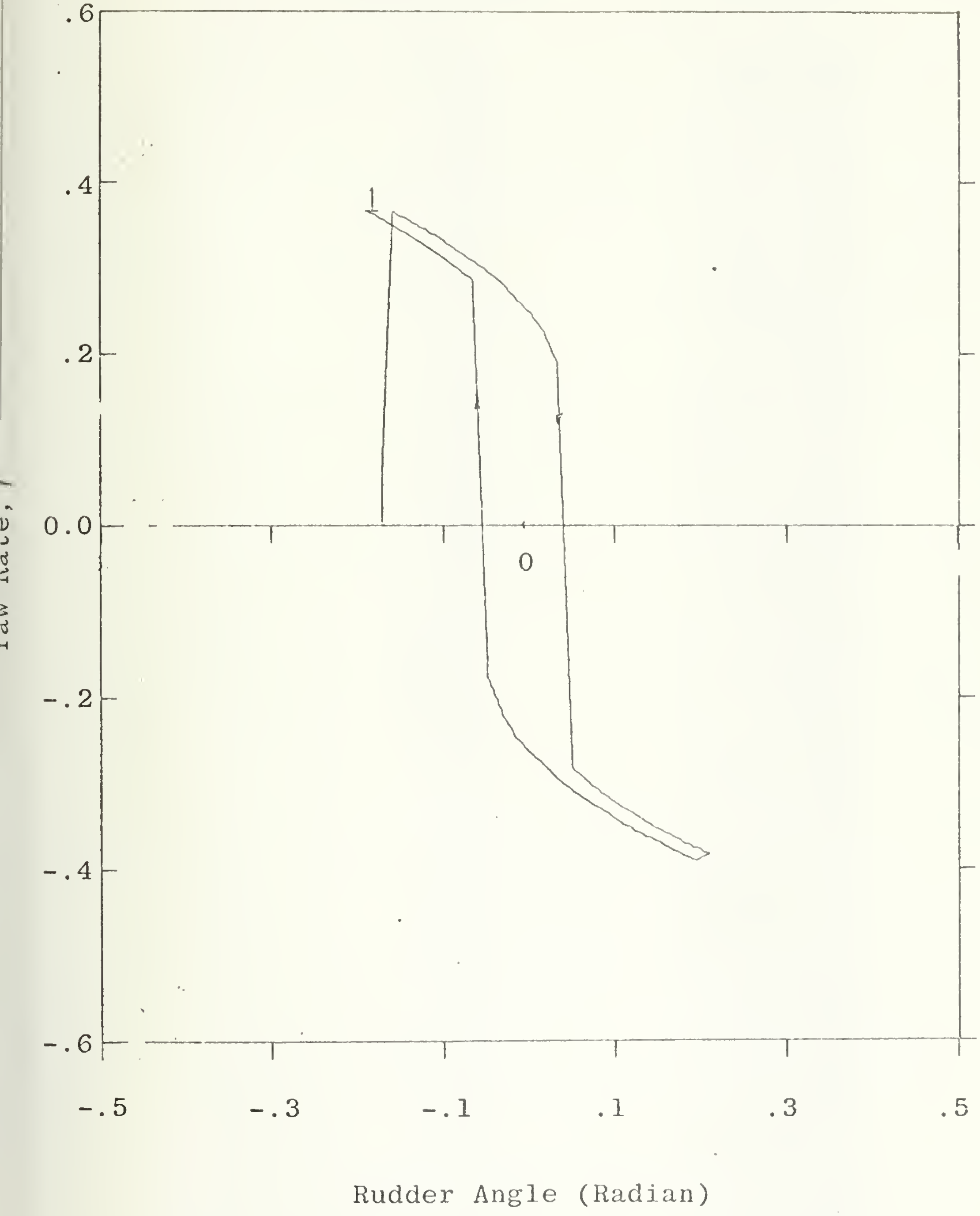


FIGURE 20. Spiral Maneuver of Model C

TABLE VII. SPIRAL MANEUVER OF MODEL C

RUDDER ANGLE (DEGREE)	YAW RATE (RAD./SEC*L/U)
*******	0.36548072
- 9.000	0.35614842
- 8.000	0.34629613
- 7.000	0.33535161
- 6.000	0.32470226
- 5.000	0.31269461
- 4.000	0.29961270
- 3.000	0.28513420
- 2.000	0.26874071
- 1.000	0.24949914
- 0.000	0.22536594
1.000	0.18941998
2.000	-0.28294164
3.000	-0.29785007
4.000	-0.31125194
5.000	-0.32350868
6.000	-0.33485866
7.000	-0.34546858
8.000	-0.35545886
9.000	-0.36492395
10.000	-0.37393415
11.000	-0.38254720
12.000	-0.39080832
11.000	-0.38356121
10.000	-0.37394869
9.000	-0.36493850
8.000	-0.35547382
7.000	-0.34548380
6.000	-0.33437481
5.000	-0.32352638
4.000	-0.31127167
3.000	-0.29787230
2.000	-0.28296709
1.000	-0.26596129
- 0.000	-0.24573880
- 1.000	-0.21966898
- 2.000	-0.17531638
- 3.000	0.28510946
- 4.000	0.29959112
- 5.000	0.31267542
- 6.000	0.32468486
- 7.000	0.33583581
- 8.000	0.34628141
- 9.000	0.35613346
*******	0.36548072

degrees port side. That means any increase of the rudder angle to port by 1.5 degrees will cause the model C to suddenly swing from starboard to port until r' = 0.28. Likewise, when the rudder angle increase from port to starboard the model C can turn against its rudder by δ= 2.5 degrees starboard and then suddenly swing in the opposite direction to a new stable position r' = 0.28.

V. CONCLUSION

This thesis has presented the characteristics of the dynamically unstable ship in stability and maneuverability.

As discussed in Part III, the most powerful factor in stabilizing a unstable ship is a lifting surface installed aft of midship; the addition of area of fin is more effective for stability than the aspect ratio. The effect of a lifting surface may be obtained by designing a fine stern with neat flow lines and deadwood or a proper rudder. It was found that the fin installed near the pivot point destabilizes the ship.

As discussed in part IV, the unstable ship is more maneuverable in a turning maneuver but less maneuverable in the standard zig-zag maneuver than the stable ship. In turning maneuver, the more unstable ship was less sensitive to the rudder deflection.

As discussed in section IV-B-1, the followings are considered necessary for optimum stability and maneuverability of a ship.

1) Locating the rudder at the point aft of the midship where the steady turning radius is minimum.

2) Installing a stabilizing fin or deadwood as the stern in order to maintain adequate stability.

DSL/360 Language was found to be very powerful for simulating ship motion.

VI. RECOMMENDATION

All results in this thesis were obtained by simulation using DSL/360 without real model tests.

The author would like to compare these results from real model tests in the future. The results in the thesis are in non-dimensional values so that they can be converted to full scale ships or models.

Also the following itmes are recommended for future study;

1) Maneuverability of the unstable ship in Z-maneuvers other than the standard Z-maneuver.

2) Maneuverability of a dynamically unstable ship with auto-steering control.

3) Ship design for the optimum stability and maneuverability.

APPENDIX A

Prediction of Hydrodynamic Coefficients in Linearized Equation of the Ship.

Using the method in Ref. 1 and 2, the following predictions can be obtained.

(1) A fin travelling at a forward velocity u and transverse velocity v has

$$(Y_v')_f = -\left| A_f' \left(\frac{\partial C_L}{\partial \beta}\right)_f \right| \tag{34}$$

where

$A_f' = A_f / LH$

A_f = area of fin

C_L = Lift Coefficient

β = Drift angle

$$\frac{\partial C_L}{\partial \beta} = \frac{2\pi}{1 + \frac{2}{a_{eff}}}$$ for general aspect ratio fins

$$= \frac{\pi}{2} a_{eff}$$ for very low aspect ratio fins

$$(N_v')_f = x_f' (Y_v')_f$$

$$(Y_r')_f = x_f' (Y_v')_f \tag{34a}$$

$$(N_r')_f = (x_f')^2 (Y_v')_f$$

$$(Y'_{\dot{v}})_f = \frac{2\pi h'_f A'_f}{(a^2+1)^{1/2}} \qquad (35)$$

where

a = aspect ratio
a_{eff} = effective aspect ratio
h'_f = h_f/L
h_f = effective span

$$\begin{aligned} (N'_{\dot{v}})_f &= x'_f\,(Y'_{\dot{v}})_f \qquad (35a)\\ (Y'_{\dot{r}})_f &= x'_f\,(Y'_{\dot{v}})_f \\ (N'_{\dot{r}})_f &= (x'_f)^2\,(Y'_{\dot{v}})_f \end{aligned}$$

Considering the rudder as a fin,

$$\begin{aligned} Y'_\delta &= -(Y'_v)_f \\ N'_\delta &= -\tfrac{1}{2}(Y'_\delta) \qquad (35b) \end{aligned}$$

(2) Considering the ship's hull as a fin with considerable drag coefficient C

$$(Y'_v)_h = -\left[\left(\frac{\partial C_L}{\partial \beta}\right)_h + (C_D)_h\right] \tag{36}$$

where
$$\left(\frac{\partial C_L}{\partial \beta}\right)_h = \frac{\pi}{2} a_{eff} = \frac{\pi H}{L}$$

$$\begin{aligned}
(N'_v)_h &= -(m'_2 - k_1 m') + x'_p (Y'_v)_h \\
(Y'_r)_h &= -k_1 m' + x'_p (Y'_v)_h \\
(N'_r)_h &= -m'_z x' + x'_0 (Y'_v)_h \\
(Y'_{\dot{v}})_h &= -m'_z \\
(N_{\dot{r}})_h &= -\frac{k'}{\frac{\rho}{2} L^4 H} \int_{bow}^{Stern} \frac{L}{H} \left(\frac{\rho}{2} \pi C_s h^2 x^2\right) dx \\
(Y'_{\dot{r}})_h &\approx 0 \\
(N'_{\dot{r}})_h &\approx 0
\end{aligned} \tag{36a}$$

(see Ref. 1 for more details)

(3) Assuming a large deadwood which has a sufficiently low aspect ratio, an effective span equal to the draft of the ship and a triangle shape, than

$$
\begin{aligned}
(Y'_v)_{f1} &= -\frac{\pi H}{L} \\
(N'_v)_{f1} &= -\frac{1}{2}(Y'_v)_{f1} \\
(Y'_r)_{f1} &= -\frac{1}{2}(Y'_v)_{f1} \\
(N'_r)_{f1} &= \frac{1}{4}(Y'_v)_{f1} \\
(Y'_{\dot{v}})_{f1} &= \frac{4\pi A'_f h'_f}{(a_{eff}+1)^{1/2}} \\
(N'_{\dot{v}})_{f1} &= -\frac{1}{2}(Y'_{\dot{v}})_{f1} \\
(Y'_{\dot{r}})_{f1} &= -\frac{1}{2}(Y'_{\dot{v}})_{f1} \\
(N'_{\dot{r}})_{f1} &= \frac{1}{4}(Y'_{\dot{v}})_{f1}
\end{aligned}
\qquad (37)
$$

(4) Assuming that a rudder or small deadwood has $\frac{\partial C_L}{\partial \beta} = \frac{2\pi}{1+\frac{2}{a_{eff}}}$ due to the high aspect ratio, then

$$\begin{aligned}
(Y'_v)_{f2} &= -A'_{f2}\left(\frac{2\pi}{1+\frac{2}{a_{eff}}}\right) \\
(N'_v)_{f2} &= -\frac{1}{2}(Y'_v)_{f2} \\
(Y'_r)_{f2} &= -\frac{1}{2}(Y'_v)_{f2} \\
(N'_r)_{f2} &= \frac{1}{4}(Y'_v)_{f2} \\
(Y'_{\dot v})_{f2} &= -\frac{4\pi A'_{f2}\,h'_{f2}}{(a_{eff}+1)^{1/2}} \\
(N'_{\dot v})_{f2} &= -\frac{1}{2}(Y'_{\dot v})_{f2} \\
(Y'_{\dot r})_{f2} &= -\frac{1}{2}(Y'_{\dot v})_{f2} \\
(N'_{\dot r})_{f2} &= \frac{1}{4}(Y'_{\dot v})_{f2}
\end{aligned} \tag{38}$$

(see Ref. 1 and 2 for more details)

APPENDIX B

Models for Stable, Marginally Unstable, Unstable, very Unstable and Extremely Unstable Ship.

In this section it is intended to obtain models for several types of unstable ships. These will be simulated using DSL/360 to study their stability and maneuverability. The original model was adopted from a five foot Series 60 model (with block coefficient 0.7 and no propeller) as originally developed by the Davidson Laboratory.

Principal characteristics of parent model.

Length (L)	5.0 ft.
Breadth (B)	0.714 ft.
Draft (H)	0.267 ft.
Block coefficient (C_B)	0.7
LCG from bow (X_G)	5.15 ft.
Displacement ($\triangle$)	41.64 lbs.
Area of rudder (A_R)	0.021 sq. ft.
Rudder span	0.2 ft.
Rudder chord	0.105 ft.
Mass (m)	1.292 slugs
Mass coeff. (m')	0.2
Longitudinal added mass coeff. (k_1m')	0.004
Lateral added mass coeff. (m'_2)	0.18

Rotational added mass coeff. (m'_2)......	0.165
Radius of gyration in air	0.25L
C. G. of lateral added mass from midship ($\frac{\bar{x}}{L}$)	0.024
Drag coeff. at zero drift angle	0.019

By using Appendix A and computer program 1, the parent model can be modified to the desired models. From the results of computer program 1, the five models were selected, which were named stable model A, marginally unstable model B, unstable model C, very unstable model D and extremely unstable model E. (See computer program 1, Appendix A and Table I)

The hydrodynamic derivatives in Table I were considered to be inherent for each model in this thesis.

APPENDIX C

Selection of the Optimum Values of the Proportionalities of the Rudder Deflection to the Ship's Heading error and Yawing Velocity.

Let the rudder deflection equation be

$$\delta(t) = k_1 \psi + k_2 \dot{\psi} \qquad \text{(see Eq. 23)}$$

The ship motion under such rudder control has the characteristic equation,

$$A s^3 + B s^2 + C s + D = 0 \qquad \text{(see Eq. 25)}$$

Let

$$s = \frac{B}{A} p$$

, then

$$\frac{B^3}{A^2} p^3 + \frac{B^3}{A^2} p^2 + \frac{CB}{A} p + D = 0$$

or $$p^3 + p^2 + \frac{AC}{B^2} p + \frac{A^2 D}{B^3} = 0$$

or $$p^3 + p^2 + b_1 p + b_0 = 0 \qquad (39)$$

where

$$b_1 = \frac{AC}{B^2} \qquad \text{and} \qquad b_0 = \frac{A^2 D}{B^3} \tag{40}$$

From Eqs. (24) and (25)

$$\begin{aligned} A &= (Y'_{\dot v} - m')(N'_{\dot r} - I'_z) - (Y'_{\dot r} - m'x'_G)(N'_{\dot v} - m'x'_G) \\ B &= c_1 k_2 + C_2 \end{aligned} \tag{41}$$

where

$$c_1 = N'_\delta (Y'_{\dot v} - m') - Y'_\delta (N'_{\dot v} - m'x_G)$$

$$c_2 = (Y'_{\dot v} - m')(N'_r - m'x'_G) + Y'_v (N'_{\dot r} - I'_z) - N'_v (Y'_{\dot r} - m'x'_G)$$

Likewise

$$C = c_3 k_1 + c_4 k_2 + c_5 \tag{42}$$

where

$$c_3 = N'_\delta (Y'_{\dot v} - m') - Y'_\delta (N'_{\dot v} - m'x'_G)$$

$$c_4 = N'_\delta Y'_v - N'_v Y'_\delta$$

$$c_5 = Y'_v (N'_r - m'x'_G) - N'_v (Y'_r - m')$$

$$D = c_6 k_1 \tag{43}$$

where

$$c_6 = (Y_v' N_\delta' - N_v' Y_\delta)$$

Eq. (41) gives

$$B^2 = c_1^2 k_2^2 + 2c_1 c_2 k_2 + c_2^2$$

$$B^3 = c_1^3 k_2^3 + 3c_1^2 c_2 k_2^2 + 3c_1 c_2^2 k_2 + c_2^3$$

Eq. (40) gives

$$\begin{aligned} b_1 &= \frac{AC}{B^2} \\ b_1(c_1^2 k_2^2 + 2c_1 c_2 k_2 + c_2^2) &= A(c_3 k_1 + c_4 k_2 + c_5) \end{aligned} \tag{44}$$

Eq. (40) gives

$$b_o = \frac{A^2 D}{B^3}$$

$$b_o(c_1^3 k_2^3 + 3c_1^2 c_2 k_2^2 + 3c_1 c_2^2 k_2 + c_2^3) = A^2 c_6 k_1$$

Thus

$$k_1 = \frac{B_o}{c_6 A^2}(c_1^3 k_2^2 + 3c_1 c_2 k_2^2 + 3c_1 c_2^2 k_2 + c_2^3) \tag{45}$$

Inserting Eq. (45) into Eq. (44)

$$B_4 k_2^3 + B_3 k_2^2 + B_2 k_2 + B_1 = 0$$

where

$$\begin{aligned} B_4 &= c_1^3 c_3 b_o / c_6 A \\ B_3 &= \frac{3c_1^2 c_2 c_3 b_o}{c_6 A} - b_1 c_1^2 \end{aligned} \tag{46}$$

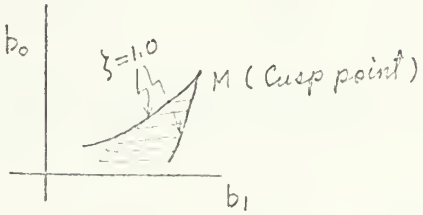
$$B_2 = \frac{3C_1 C_2^2 C_3 b_0}{C_6 A} + A C_4 - 2 b_1 C_1 C_2$$

$$B_1 = \frac{C_2^3 C_3 b_0}{C_6 A} + A C_5 - b_1 C_2^2$$

In Eq. (46) all coefficients except b_o and b_1 can be obtained from hydrod-namic derivatives of a given ship.

Calculator of b_o and b_1

For 3 real negative roots of the characterisitic equator, $As^3 + Bs^2 + Cs + D = 0$, the point M on Mitrovics chart (see Ref. 7) should be in $\zeta = 1.0$ region



For 3 repeated roots, M should be the cusp point.

At the cusp point,

$$\frac{db_o}{db_1} = \text{undefined}$$

$$\frac{db_o}{db_1} = \frac{db_o/d\omega_t}{db_1/d\omega_t}.$$

Therefore

$$\frac{db_o}{d\omega_t} = 0 \quad \text{and} \quad \frac{db_1}{d\omega_t} = 0$$

For the third order characteristic Eq. with = 1.0

$$b_o = \omega_t^2 - 2\omega_t^2$$

$$b_1 = 2\omega_t - 3\omega_t^2$$

$$\frac{db_1}{d\omega_t} = 2 - 6\omega_t = 0$$

or $\omega_t = \frac{1}{3}$

So, $b_1 = \frac{1}{3}$ and $b_o = \frac{1}{27}$

(see Ref. 7 for more details)

Note: It is difficult to find the exact cusp point by the digital computer due to the non-integer values of b_o and b_1.

```
                   < COMPUTER PROGRAM #1 >
*** SELECTION OF THE DYNAMICALLY UNSTABLE MODELS ***

* THIS WAS PROGRAMMED TO SELECT THE UNSTABLE MODELS BY MODIFYING ***
* THE 5 FOOT  SERIES 60 MODEL OF DAVIDSON LAB. ***

* REFERENCE ; APPENDIX A AND B


//PARK1851 JOB (2595,0286FT,NH22),'PARK',TIME=1
// EXEC DSL4,REGION=150K
//DSL.INPUT DD *

LABEL CALCULATION OF HYDRODYNAMIC DERIVATIVES OF  SHIP'S MODEL

INTGER NUM,NPLOT
INTEG RKSFX

*      MODEL DIMENSIONS

CONST DRAFT=.267,LBP=5.,CDH=.0190,YADMAS=.18,XP=.011,MASS=.2,K1=.022,...
      XBAR=.024,XO=.307,ZADMAS=.165,IZ=.0125,XG=-.015,UO=1.
CONST NPLOT=10

*      FIN DIMENSIONS

CONST EARO=1.5,AFO=.0    ,HFO=.03

*      RUDDER DIMENSIONS

CONST EARR=3.8,AFR=.0   ,HFR=.04

*      DEADWOOD DIMENSIONS

CONST EAR1=2.5,AF1=.032,HF1=.0534

CONST XF=-0.5
INITIAL
      NUM=0

*      CALCULATION OF THE HYDRODYNAMIC DERIVATIVES FOR THE BARE HULL

      YVH=-3.14*DRAFT/LBP-CDH
      NVH=-1.*(YADMAS-K1*MASS)+XP*YVH
      YRH=-K1*MASS+XP*YVH
      NRH=-ZADMAS*XBAR+XO*XO*YVH
```

```
      YVDTH=-YADMAS
      NVDTH=0.
      YRDTH=0.
      NRDTH=IZ-0.0237
*     CALCULATION OF THE HYDRODYNAMIC DERIVATIVES FOR THE MOVABLE FIN

      CLBFO=2.*3.14/(1.+2./EARO)
      YVFO=-AFO*CLBFO
      NVFO=YVFO*XF
      YRFO=YVFO*XF
      NRFO=YVFO*XF**2
      YVDTFO=-4.*3.14*HFO*AFO/(EARO**2+1.)**.5
      NVDTFO=YVDTFO*XF
      YRDTFO=YVDTFO*XF
      NRDTFO=YVDTFO*XF**2
*     CALCULATION OF THE HYDRODYNAMIC DERIVATIVES FOR THE RUDDER

      YVFR=-AFR*2.*3.14/(1.+2./EARR)
      NVFR=-0.5*YVFR
      YRFR=-0.5*YVFR
      NRFR=0.25*YVFR
      YVDTFR=-4.*3.14*HFR*AFR/(EARR  **2+1.)**0.5
      NVDTFR=-0.5*YVDTFR
      YRDTFR=-0.5*YVDTFR
      NRDTFR=0.25*YVDTFR
*     CALCULATION OF THE HYDRODYNAIC DERIVATIVES FOR LARGE DEADWOOD
      INCLUDING RUDDER

      YVF1=-3.14*DRAFT/LBP
      NVF1=-0.5*YVF1
      YRF1=-0.5*YVF1
      NRF1=0.25*YVF1
      YVDTF1=-4.*3.14*HF1*AF1/(EAR1  **2+1.)**0.5
      NVDTF1=-0.5*YVDTF1
      YRDTF1=-0.5*YVDTF1
      NRDTF1=0.25*YVDTF1
*     CALCULATION OF THE HYDRODYNAMIC DERIVATIVES FOR
*     THE BARE HULL WITH RUDDER AND MOVABLE FIN BUT WITHOUT DEADWOOD

      YVO=YVH+YVFO+YVFR
      NVO=NVH+NVFO+NVFR
      YRO=YRH+YRFO+YRFR
      NRO=NRH+NRFO+NRFR
      YVDTO=YVDTH+YVDTFO+YVDTFR
```

```
      NVDTO=NVDTH+NVDTFO+NVDTFR
      YRDTO=YRDTH+YRDTFO+YRDTFR
      NRDTO=NRDTH+NRDTFO+NRDTFR
      AO=(MASS-YVDTO)*(IZ-NRDTO)-(YRDTO-MASS*XG)*(NVDTO-MASS*XG)
      BO=-((MASS-YVDTO)*(NRO-MASS*XG*UO)+YVO*(IZ-NRDTO)+NVO*(YRDTO-...
        MASS*XG)+(YRO-MASS*UO)*(NVDTO-MASS*XG))
      CO=YVO*(NRO-MASS*XG*UO)-NVO*(YRO-MASS*UO)
      SGMAO1=(-BO/AO+SQRT((BO/AO)**2-4.*CO/AO))*0.5
      SGMAO2=(-BO/AO-SQRT((BO/AO)**2-4.*CO/AO))*0.5
      VO1=(SGMAO1-SGMAO2)/AO
      VO2=-VO1
      WRITE(6,10) YVO,NVO,YRO,NRO,YVDTO,NVDTO,YRDTO,NRDTO,XF,AFO,AFR,...
       SGMAO1

*     CALCULATION OF THE HYDRODYNAMIC DERIVATIVES FOR
*     THE BARE HULL WITH RUDDER AND MOVABLE FIN WITH DEADWOOD

      YV1=YVH+YVFO+YVF1
      NV1=NVH+NVFO+NVF1
      YR1=YRH+YRFO+YRF1
      NR1=NRH+NRFO+NRF1
      YVDT1=YVDTH+YVDTFO+YVDTF1
      NVDT1=NVDTH+NVDTFO+NVDTF1
      YRDT1=YRDTH+YRDTFO+YRDTF1
      NRDT1=NRDTH+NRDTFO+NRDTF1
      A1=(MASS-YVDT1)*(IZ-NRDT1)-(YRDT1-MASS*XG)*(NVDT1-MASS*XG)
      B1=-((MASS-YVDT1)*(NR1-MASS*XG*UO)+YV1*(IZ-NRDT1)+NV1*(YRDT1-...
        MASS*XG)+(YR1-MASS*UO)*(NVDT1-MASS*XG))
      C1=YV1*(NR1-MASS*XG*UO)-NV1*(YR1-MASS*UO)
      SGMA11=(-B1/A1+SQRT((B1/A1)**2-4.*C1/A1))*0.5
      SGMA12=(-B1/A1-SQRT((B1/A1)**2-4.*C1/A1))*0.5
      V11=(SGMA11-SGMA12)/A1
      V12=-V11
      WRITE(6,10) YV1,NV1,YR1,NR1,YVDT1,NVDT1,YRDT1,NRDT1,XF,AFO,AF1,...
       SGMA11
   10 FORMAT(' ',13F10.5)
DERIVATIVE
      VO=VO1*EXP(SGMAO1*TIME)+VO2*EXP(SGMAO2*TIME)
      V1=V11*EXP(SGMA11*TIME)+V12*EXP(SGMA12*TIME)
CONTRL FINTIM=1.0,DELT=0.005,DELS=0.01
PRINT SGMA
SAMPLE
      NUM=NUM+1
      CALL DRWG(1,1,NUM,TIME,VO)
      CALL DRWG(1,2,NUM,TIME,V1)
TERMINAL
      CALL ENDRW(NPLOT)
      GO TO (1,2,3,4,5.6,7,8,9),NPLOT
```

```
    9 AFR=.016
      GO TO 11
    8 AFO=.0225
      GO TO 11
    7 XF=-0.3
      GO TO 11
    6 XF=0.0
      GO TO 11
    5 XF=0.1
      GO TO 11
    4 XF=0.2
      GO TO 11
    3 XF=0.3
      GO TO 11
    2 XF=0.4
      GO TO 11
    1 XF=0.5
   11 CALL RERUN
END
STOP
//PLOT.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//     DCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT.SYSIN DD *
SELECTION OF MODEL
D.S.PARK   181115                                                               05
5         0.625     0.        1          8         8
```

< COMPUTER PROGRAM #2 >

```
***EVALUATION OF THE EFFECT OF FIN LOCATION AND SIZE ***

//PARK1852 JOB (2595,0286FT,NH22),'PARK',TIME=4
// EXEC DSL4,REGION=150K
//DSL.INPUT DD *
LABEL EVALUATION OF THE EFFECT OF FIN LOCATION AND SIZE
INTGER NUM,NPLOT,I
INTEG RKSFX
CONST NPLOT=7
CONST DRAFT=.267,LBP=5.,CDH=.0190,YADMAS=.18,XP=.011,MASS=.2,K1=.022,...
      XBAR=.024,XO=.307,ZADMAS=.165,IZ=.0125,XG=-.015,UO=1.
CONST EARO=1.5,AFO=.0225,HFO=.03
INITIAL
      NUM=0
      CLBFO=2.*3.14/(1.+2./EARO)

* VECTOR ARRAY; 1;MODEL A
                2;MODEL B
                3;MODEL C
                4;MODEL D
                5;MODEL E

STORAG YVH(5),NVH(5),YRH(5),NRH(5),YVDTH(5),NVDTH(5),YRDTH(5),...
       NRDTH(5),YV(5),NV(5),YR(5),NR(5),YVDT(5),NVDT(5),YRDT(5),...
       NRDT(5),A(5),B(5),C(5),SGMA(5)
TABLE YVH  (1-5)=-0.35435,-0.31306,-0.31306,-0.31306,-0.18668,...
      NVH  (1-5)=-0.09382,-0.11446,-0.12657,-0.14474,-0.17765,...
      YRH  (1-5)=+0.07738,+0.05674,+0.04463,+0.02646,-0.00645,...
      NRH  (1-5)=-0.06347,-0.05315,-0.04346,-0.03801,-0.02155,...
      YVDTH(1-5)=-0.18797,-0.18675,-0.18675,-0.18675,-0.18000,...
      NVDTH(1-5)=+0.00399,+0.00377,+0.00243,+0.00102,+0.00000,...
      YRDTH(1-5)=+0.00399,+0.00377,+0.00243,+0.00102,+0.00000,...
      NRDTH(1-5)=-0.01319,-0.01289,-0.01213.-0.01171,-0.01120
      WRITE(6,2000)
 2000 FORMAT(4X,'XF',8X,'SIGMA1',4X,'SIGMA2',4X,'SIGMA3',4X,
     1'SIGMA4',4X,'SIGMA5')
DERIVATIVE

* CALCULATION OF HYDRODYNAMIC DERIVATIVES OF MOVING FIN

      XF=TIME-0.5
      YVFO=-AFO*CLBFO
      NVFO=YVFO*XF
      YRFO=YVFO*XF
      NRFO=YVFO*XF**2
```

```
          YVDTFO=-4.*3.14*HFO*AFO/(EARO**2+1.)**0.5
          NVDTFO=YVDTFO*XF
          YRDTFO=YVDTFO*XF
          NRDTFO=YVDTFO*XF**2
DYNAMIC

* CALCULATION OF HYDRODYNAMIC DERIVATIVES OF SHIP WITH FIN

          DO 1000 I=1,5
          YV(I)=YVH(I)+YVFO
          NV(I)=NVH(I)+NVFO
          YR(I)=YRH(I)+YRFO
          NR(I)=NRH(I)+NRFO
          YVDT(I)=YVDTH(I)+YVDTFO
          NVDT(I)=NVDTH(I)+NVDTFO
          YRDT(I)=YRDTH(I)+YRDTFO
          NRDT(I)=NRDTH(I)+NRDTFO
          A(I)=(MASS-YVDT(I))*(IZ-NRDT(I))-(YRDT(I)-MASS*XG)*(NVDT(I)-...
                  MASS*XG)
          B(I)=-((MASS-YVDT(I))*(NR(I)-MASS*XG*UO)+YV(I)*(IZ-NRDT(I))+...
                  NV(I)*(YRDT(I)-MASS*XG)+(YR(I)-MASS*UO)*(NVDT(I)-MASS*XG))
          C(I)=YV(I)*(NR(I)-MASS*XG*UO)-NV(I)*(YR(I)-MASS*UO)
          SGMA(I)=(-B(I)/A(I)+SQRT((B(I)/A(I))**2-4.*C(I)/A(I)))*0.5
     1000 CONTINUE
          WRITE(6,3000) XF,SGMA
     3000 FORMAT(2X,F5.3,4X,5F10.5)
CONTRL  FINTIM=1.0,DELT=0.01,DELS=0.01
SAMPLE
          NUM=NUM+1
          CALL  DRWG(1,1,NUM,XF,SGMA(1))
          CALL  DRWG(1,2,NUM,XF,SGMA(2))
          CALL  DRWG(1,3,NUM,XF,SGMA(3))
          CALL  DRWG(1,4,NUM,XF,SGMA(4))
          CALL  DRWG(1,5,NUM,XF,SGMA(5))
PRINT XF
TERMINAL
          CALL ENDRW(NPLOT)
          GO TO (1,2,3,4,5,6),NPLOT
        6 HFO=.0225
          AFO=.01265
          GO TO 12
        5 HFO=0.015
          AFO=0.0056
          GO TO 12
        4 HFO=.045
          AFO=.0506
          GO TO 12
        3 EARO=2.
```

```
      HFO=0.0173
      AFO=.0056
      GO TO 12
    2 EARO=3.
      HFO=0.0212
      GO TO 12
    1 EARO=4.
      HFO=0.02448
   12 CALL RERUN
END
STOP
//PLOT.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//      DCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT.SYSIN DD *
                                        D.S.PARK                      05
-0.5      0.2       -1.5      0.375     5.        8.
                                        D.S.PARK                      05
-0.5      0.2       -1.5      0.375     5.        8.
                                        D.S.PARK                      05
-0.5      0.2       -1.5      0.375     5.        8.
                                        D.S.PARK                      05
-0.5      0.2       -1.5      0.375     5.        8.
                                        D.S.PARK                      05
-0.5      0.2       -1.5      0.375     5.        8.
                                        D.S.PARK                      05
-0.5      0.2       -1.5      0.375     5.        8.
                                        D.S.PARK                      05
-0.5      0.2       -1.5      0.375     5.        8.
```

< COMPUTER PROGRAM #3 >

```
*** EVALUATION OF THE EFFECT OF THE CENTER OF GRAVITY  ***
***  OF THE SHIP ON STABILITY***


//PARK1853 JOB (2595,0286FT,NH22),'PARK',TIME=1
// EXEC DSL4,REGION=150K
//DSL.INPUT DD *
INTGER NUM,NPLOT,NOCV
INTEG RKSFX
CONST NPLOT=1
INITIAL
      NUM=0

* HYDRODYNAMIC DERIVATIVES OF MODEL C

CONST YVO=-0.31306,NVO=-0.12657,YRO=0.04463,NRO=-.04346,...
      YVDTO=-.18675,NVDTO=.00243,YRDTC=.00243,NRDTC=-.01213
CONST           MASS=.2,IZ=.18,YD=.0658,ND=-.0329,XG=-.015,UO=1.
DERIVATIVE
      XG=-0.4+TIME
      AO=(MASS-YVDTO)*(IZ-NRDTO)-(YRDTO-MASS*XG)*(NVDTO-MASS*XG)
      BO=-((MASS-YVDTO)*(NRO-MASS*XG*UO)+YVO*(IZ-NRDTO)+NVO*(YRDTO-...
        MASS*XG)+(YRO-MASS*UO)*(NVDTO-MASS*XG))
      CO=YVO*(NRO-MASS*XG*UO)-NVO*(YRO-MASS*UO)
      SGMAO1=(-BO/AO-SQRT((BO/AO)**2-4.*CO/AO))*0.5
      SGMAO2=(-BO/AO+SQRT((BO/AO)**2-4.*CO/AO))*0.5
SAMPLE
      NUM=NUM+1
      CALL DRWG(1,1,NUM,XG,SGMAO2)
CONTRL FINTIM=.8,DELT=0.001,DELS=0.001
PRINT 0.05,XG,YVO,NVO,YRO,NRO,CO,SGMAO2
PREPAR 0.05,XG,SGMAO2
GRAPH XG,SGMAO2
PRPLOT XG,SGMAO2
TERMINAL
      CALL ENDRW(NPLOT)
END
STOP
//PLCT.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//     DCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT.SYSIN DD *
                                     D.S.PARK                               05
-0.5      0.2       -0.4      0.2      5.        6.
```

```
                 < COMPUTER PROGRAM #4 >
*** STABILIZATION OF THE UNSTABLE SHIP  BY AUTO-STEERING CONTROL ***

* REFERENCE; APPENDIX C  ***


//PARK1854 JOB (2595,0286FT,NH22),'PARK',TIME=1
// EXEC DSL4,REGION=150K
//DSL.INPUT DD *
LABEL  STABILIZATION OF UNSTABLE SHIP BY AUTO STEERING CONTROL
INTGER NUM,NPLOT,IC,IR,I,J
INTEG RKSFX
CONST NPLOT=1
INITIAL
       NUM=0
STORAG A(4),Q(4),E(4),POL(4),K2(4) B(4),IM(4)
CONST  YVO=-0.31306,NVO=-0.12657,YRO=0.04463,NRO=-.04346,...
       YVDTO=-.18675,NVDTO=.00243,YRDTC=.00243,NRDTC=-.01213
CONST  IC=4,IR=3,MASS=.2,IZ=.18,YD=.0658,ND=-.0329,XG=-.015,UO=1.
       A3=(YVDTO-MASS)*(NRDTO-IZ)-(YRDTO-MASS*XG)*(NVDTO-MASS*XG)
       B1=1./3.
       C1=ND*(YVDTO-MASS)-YD*(NVDTO-MASS*XG)
       C2=(YVDTO-MASS)*(NRO-MASS*XG)+YVO*(NRDTO-IZ)-NVO*(YRDTO-MASS*XG...
          )-(YRO-MASS)*(NVDTO-MASS*XG)
       C3=ND*(YVDTO-MASS)-YD*(NVDTO-MASS*XG)
       C4=YVO*ND-NVO*YD
       C5=YVO*(NRO-MASS*XG)-NVO*(YRO-MASS)
       C6=YVO*ND-NVO*YD
       DO 3000 J=1,40
       WRITE(6,1000) J
 1000  FORMAT(//1X,'J= ',I2)
       BC=0.037035000-0.00000001*J
       B(1)=C2**3*C3*BO/(C6*A3)+A3*C5-B1*C2**2
       B(2)=3*C1*C2**2*C3*BO/C6/A3+A3*C4-2*B1*C1*C2
       B(3)=3*C1**2*C2*C3*BO/C6/A3-B1*C1**2
       B(4)=C1**3*C3*BO/C6/A3
       CALL PRQD(B,IC,K2,IM,PCL,IR,IER)
       IF (IM(I).EQ.0.) KK2=K2(I)
       KK1=BO/C6/A3**2*(C1**3*KK2**3+3*C2*C1**2*KK2**2+3*C1*C2**2*KK2...
          +C2**3)
       A(4)=A3
       A(3)=(YVDTO-MASS)*(NRO-MASS*XG*UO+KK2*ND)+YVO*(NRDTO-IZ)-...
       NVO*(YRDTO-MASS*XG)-(YRO-MASS*UO+KK2*YD)*(NVDTO-MASS*XG)
       A(2)=(YVDTO-MASS)*(KK1*ND)+YVO*(NRO-MASS*XG*UO+KK2*ND)-...
       NVO*(YRO-MASS*UO+KK2*YD)-KK1*YD*(NVDTO-MASS*XG)
       A(1)=+KK1*YVO*ND-KK1*YD*NVO
       CALL PRQD(A,IC.Q,E,PCL;IR,IER)
```

```
      DO 1500 I=1,3
      WRITE(6,5000)I,Q(I),I,E(I)
 5000 FORMAT(1X,'Q(',I1,')= ',F10.5,10X,'E(',I1,')= ',F10.5)
 1500 CONTINUE
      IF(ABS(E(2)).LT.0.0010) GO TO 4000
 3000 CONTINUE
 4000 S1=Q(1)
      S2=Q(2)
      S3=Q(3)
      WRITE(6,2000)KK1,KK2
 2000 FORMAT(//1X,'KK1= ',F10.5,10X,'KK2= ',F10.5)
DERIVATIVE
      PSI=0.1*(EXP(S1*TIME)+EXP(S2*TIME)+EXP(S3*TIME))
SAMPLE
      NUM=NUM+1
      CALL DRWG(1,1,NUM,TIME,PSI)
CONTRL FINTIM=15.,DELT=0.01,DELS=0.02
PRINT 0.1,PSI
TERMINAL
      CALL ENDRW(NPLOT)
END
STOP
//PLOT.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//      DCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT.SYSIN DD *

                                         D.S.PARK                            05
0.        2.        0.        0.1        8.        7.
```

< COMPUTER PROGRAM #5 >

```
*** TURNING TEST FOR COMPARISON OF THE UNSTABLE SHIP WITH STABLE SHIP ***


*  SELECTED RUDDER ANGLE (DMAX): 10,20 AND 35 DEGREE
   ANGULAR VELOCITY OF RUDDER=2 DEGREE PER SEC FCR PROTYPE SHIP
   TIME DELAY OF STEERING SYSTEM=0.

//PARK1855 JOB (2595,0286FT,NH22),'PARK',TIME=30
// EXEC DSL4,REGION=150K
//DSL.INPUT DD *
INTGER NUM,NPLOT
INTEG RKSFX
CONST NPLOT=3

* HYCRODYNAMIC DERIVATIVES OF MODEL A

CONST YVA=-.35435,NVA=-.09382,YRA=.07738,NRA=-.06347,...
      YVDTA=-.18797,NVDTA=.00399,YRDTA=.00399,NRDTA=-.01319

* HYCRODYNAMIC DERIVATIVES OF MODEL B

CONST YVB=-.31306,NVB=-.11446,YRB=.05674,NRB=-.05315,...
      YVDTB=-.18675,NVDTB=.00337,YRDTB=.00337,NRDTB=-.01289

* HYDRODYNAMIC DERIVATIVES OF MODEL C

CONST YVC=-.31306,NVC=-.12657,YRC=.04463,NRC=-.04346,...
      YVDTC=-.18675,NVDTC=.00243,YRDTC=.00243,NRDTC=-.01213

* HYCRODYNAMIC DERIVATIVES OF MODEL D

CONST YVD=-.31306,NVD=-.14474,YRD=.02646.NRD=-.03801,...
      YVDTD=-.18675,NVDTD=.00102,YRDTD=.00102,NRDTD=-.01171

* HYDRODYNAMIC DERIVATIVES OF MODEL E

CONST YVE=-.18668,NVE=-.17765,YRE=-.00645,NRE=-.02155,...
      YVDTE=-.18,NVDTE=0.,YRDTE=0.,NRDTE=-.0112
CONST YVVV=-12.9,YRRR=-.2766,YRVV=-.2,NVVV=-2.31,...
      NVRR=-.612,NRRR=-.606,NRVV=-2.84,YVRR=-2.36
CONST IZ=.0125,MASS=.2,XG=-.015,Y0=.00016,N0=-.0003,...
      ND=-.03293,YD=.06585,U0=1.,V0=0.,R0=0.,XAXC=0.,YAX0=0.
CONST DMAX=35.
 INITIAL
      NUM=0
      LSHIP=500.
```

```
      LMODEL=5.
      USHIP=15.*1.6878
      UMODEL=1.5*1.6878
      PSIO=0.0
      TLAG=0.
      TDMAX=TLAG+DMAX
      DETA=(MASS-YVDTA)*(IZ-NRDTA)-(MASS*XG-NVDTA)*(MASS*XG-YRDTA)
      DETB=(MASS-YVDTB)*(IZ-NRDTB)-(MASS*XG-NVDTB)*(MASS*XG-YRDTB)
      DETC=(MASS-YVDTC)*(IZ-NRDTC)-(MASS*XG-NVDTC)*(MASS*XG-YRDTC)
      DETD=(MASS-YVDTD)*(IZ-NRDTD)-(MASS*XG-NVDTD)*(MASS*XG-YRDTD)
      DETE=(MASS-YVDTE)*(IZ-NRDTE)-(MASS*XG-NVDTE)*(MASS*XG-YRDTE)
      DRATE=40.
DERIVATIVE
      TMODEL=TIME*LMODEL/UMODEL
      TSHIP=TIME*LSHIP/USHIP
      D=-DRATE*(RAMP(TLAG)-RAMP(TDMAX/DRATE))/57.3
      U=INTGRL(UO,UDOT)
      VA=INTGRL(VO,VDOTA)
      RA=INTGRL(RO,RDOTA)
      PSIA=INTGRL(PSIO,RA)
      F2A=YO+YVA*VA+1./6.*YVVV*VA**3+1./2.*YVRR*VA*RA**2+(YRA-MASS)*...
            RA+1./6.*YRRR*RA**3+.5*YRVV*RA*VA**2+YD*D
      F3A=NO+NVA*VA+1./6.*NVVV*VA**3+.5*NVRR*VA*RA**2+(NRA-MASS*XG)...
            *RA+1./6.*NRRR*RA**3+NRVV*RA*VA**2*.5+ND*D
      NMVDTA=(IZ-NRDTA)*F2A-(MASS*XG-YRDTA)*F3A
      NMRDTA=(MASS-YVDTA)*F3A-(MASS*XG-NVDTA)*F2A
      VDOTA=NMVDTA/DETA
      RDOTA=NMRDTA/DETA
      XAXDTA=U*COS(PSIA)-VA*SIN(PSIA)
      YAXDTA=U*SIN(PSIA)+VA*COS(PSIA)
      XAXA=INTGRL(XAXO,XAXDTA)
      YAXA=INTGRL(YAXO,YAXDTA)
      VB=INTGRL(VO,VDOTB)
      RB=INTGRL(RO,RDOTB)
      PSIB=INTGRL(PSIO,RB)
      F2B=YO+YVB*VB+1./6.*YVVV*VB**3+1./2.*YVRR*VB*RB**2+(YRB-MASS)*...
            RB+1./6.*YRRR*RB**3+.5*YRVV*RB*VB**2+YD*D
      F3B=NO+NVB*VB+1./6.*NVVV*VB**3+.5*NVRR*VB*RB**2+(NRB-MASS*XG)...
            *RB+1./6.*NRRR*RB**3+NRVV*RB*VB**2*.5+ND*D
      NMVDTB=(IZ-NRDTB)*F2B-(MASS*XG-YRDTB)*F3B
      NMRDTB=(MASS-YVDTB)*F3B-(MASS*XG-NVDTB)*F2B
      VDOTB=NMVDTB/DETB
      RDOTB=NMRDTB/DETB
      XAXDTB=U*COS(PSIB)-VB*SIN(PSIB)
      YAXDTB=U*SIN(PSIB)+VB*COS(PSIB)
      XAXB=INTGRL(XAXO,XAXDTB)
      YAXB=INTGRL(YAXO,YAXDTB)
      VC=INTGRL(VO,VDOTC)
```

```
      RC=INTGRL(RO,RDOTC)
      PSIC=INTGRL(PSIO,RC)
      F2C=YO+YVC*VC+1./6.*YVVV*VC**3+1./2.*YVRR*VC*RC**2+(YRC-MASS)*...
            RC+1./6.*YRRR*RC**3+.5*YRVV*RC*VC**2+YD*D
      F3C=NO+NVC*VC+1./6.*NVVV*VC**3+.5*NVRR*VC*RC**2+(NRC-MASS*XG)...
            *RC+1./6.*NRRR*RC**3+NRVV*RC*VC**2*.5+ND*D
      NMVDTC=(IZ-NRDTC)*F2C-(MASS*XG-YRDTC)*F3C
      NMRDTC=(MASS-YVDTC)*F3C-(MASS*XG-NVDTC)*F2C
      VDOTC=NMVDTC/DETC
      RDOTC=NMRDTC/DETC
      XAXDTC=U*COS(PSIC)-VC*SIN(PSIC)
      YAXDTC=U*SIN(PSIC)+VC*COS(PSIC)
      XAXC=INTGRL(XAXO,XAXDTC)
      YAXC=INTGRL(YAXO,YAXDTC)
      VD=INTGRL(VO,VDOTD)
      RD=INTGRL(RO,RDOTD)
      PSID=INTGRL(PSIO,RD)
      F2D=YO+YVD*VD+1./6.*YVVV*VD**3+1./2.*YVRR*VD*RD**2+(YRD-MASS)*...
            RD+1./6.*YRRR*RD**3+.5*YRVV*RD*VD**2+YD*D
      F3D=NO+NVD*VD+1./6.*NVVV*VD**3+.5*NVRR*VD*RD**2+(NRD-MASS*XG)...
            *RD+1./6.*NRRR*RD**3+NRVV*RD*VD**2*.5+ND*D
      NMVDTD=(IZ-NRDTD)*F2D-(MASS*XG-YRDTD)*F3D
      NMRDTD=(MASS-YVDTD)*F3D-(MASS*XG-NVDTD)*F2D
      VDOTD=NMVDTD/DETD
      RDOTD=NMRDTD/DETD
      XAXDTD=U*COS(PSID)-VD*SIN(PSID)
      YAXDTD=U*SIN(PSID)+VD*COS(PSID)
      XAXD=INTGRL(XAXO,XAXDTD)
      YAXD=INTGRL(YAXO,YAXDTD)
      VE=INTGPL(VO,VDOTE)
      RE=INTGRL(RO,RDOTE)
      PSIE=INTGRL(PSIO,RE)
      F2E=YO+YVE*VE+1./6.*YVVV*VE**3+1./2.*YVRR*VE*RE**2+(YRE-MASS)*...
            RE+1./6.*YRRR*RE**3+.5*YRVV*RE*VE**2+YD*D
      F3E=NO+NVE*VE+1./6.*NVVV*VE**3+.5*NVRR*VE*RE**2+(NRE-MASS*XG)...
            *RE+1./6.*NRRR*RE**3+NRVV*RE*VE**2*.5+ND*D
      NMVDTE=(IZ-NRDTE)*F2E-(MASS*XG-YRDTE)*F3E
      NMRDTE=(MASS-YVDTE)*F3E-(MASS*XG-NVDTE)*F2E
      VDOTE=NMVDTE/DETE
      RDOTE=NMRDTE/DETE
      XAXDTE=U*COS(PSIE)-VE*SIN(PSIE)
      YAXDTE=U*SIN(PSIE)+VE*COS(PSIE)
      XAXE=INTGRL(XAXO,XAXDTE)
      YAXE=INTGRL(YAXO,YAXDTE)
SAMPLE
      NUM=NUM+1
      CALL DRWG(1,1,NUM,YAXA,XAXA)
      CALL DRWG(1,2,NUM,YAXB,XAXB)
```

```
      CALL DRWG(1,3,NUM,YAXC,XAXC)
      CALL DRWG(1,4,NUM,YAXD,XAXD)
      CALL DRWG(1,5,NUM,YAXE,XAXE)
CONTRL FINTIM=39.,DELT=0.01,DELS=0.02
PRINT .2,XAXA,XAXB,XAXC,XAXE,YAXA,YAXB,YAXC,YAXD,YAXE
TERMINAL
      CALL ENDRW(NPLOT)
      GO TO(1,2),NPLOT
    2 DMAX=20.
      GO TO 11
    1 DMAX=10
   11 CALL RERUN
END
STOP
//PLOT.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//       DCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT.SYSIN DD *
                                        D.S.PARK                             05
-0.5      1.5       -2.5      1.5       6.        9.
                                        D.S.PARK                             05
-0.5      1.5       -2.5      1.5       6.        9.
                                        D.S.PARK                             05
-0.4      2.        -4.       2.        6.        9.
```

< COMPUTER PROGRAM #6 >

*** EVALUATION OF THE EFFECT OF RUDDER LOCATION ON MANEUVERABILITY ***

```
//PARK1856 JOB (2595,0286FT,NH22),'PARK',TIME=8
// EXEC DSL4,REGION=150K
//DSL.INPUT DD *
INTGER NUM,NPLOT
INTEG RKSFX
CONST NPLOT=9
CONST YVH=-.31306,NVH=-.12657,YRH=.04463,NRH=-.04346,...
      YVDTH=-.18675,NVDTH=.00243,YRDTH=.00243,NRDHC=-.01213
CONST YVVV=-12.9,YRRR=-.2766,YRVV=-.2,NVVV=-2.31,...
      NVRR=-.612,NRRR=-.606,NRVV=-2.84,YVRR=-2.36
CONST IZ=.0125,MASS=.2,XG=-.015,YO=.00016,NO=-.0003,...
      ND=-.03293,YD=.06585,UO=1.,VO=0.,RO=0.,XAXO=0.,YAXO=0.
CONST EARR=3.8,AFR=0.016,HFR=0.04
CONST XF=0.5
INITIAL
      NUM=0
      XU=-.0253
      XUDOT=0.025
      PSIO=0.
      LSHIP=500.
      LMODEL=5.
      USHIP=15.*1.6878
      UMODEL=1.5*1.6878
      XAXO=0.
      YAXO=0.
      DMAX=20.
      TLAG=0.
      TDMAX=TLAG+DMAX
      F1=0.
      EARO=EARR
      AFO=AFR
      HFO=HFR
      YVFR=0.
      NVFR=-0.5*YVFR
      YRFR=-0.5*YVFR
      NRFR=0.25*YVFR
      YVDTFR=0.
      NVDTFR=-0.5*YVDTFR
      YRDTFR=-0.5*YVDTFR
      NRDTFR=0.25*YVDTFR
      CLBFO=2.*3.14/(1.+2./EARO)
      NDP=YD*XF
      YVFO=-AFO*CLBFO
```

```
      NVFO=YVFO*XF
      YRFO=YVFO*XF
      NRFO=YVFO*XF**2
      YVDTFO=-4.*3.14*HFO*AFO/(EARO**2+1.)**0.5
      NVDTFO=YVDTFO*XF
      YRDTFO=YVDTFO*XF
      NRDTFO=YVDTFO*XF**2
      YVC=YVH+YVFO-YVFR
      NVC=NVH+NVFO-NVFR
      YRC=YRH+YRFO-YRFR
      NRC=NRH+NRFO-NRFR
      YVDTC=YVDTH+YVDTFO-YVDTFR
      NVDTC=NVDTH+NVDTFO-NVDTFR
      YRDTC=YRDTH+YRDTFO-YRDTFR
      NRDTC=NRDTH+NRDTFO-NRDTFR
      DETC=(MASS-YVDTC)*(IZ-NRDTC)-(MASS*XG-NVDTC)*(MASS*XG-YRDTC)
      DRATE=40.
      AC=(MASS-YVDTC)*(IZ-NRDTC)-(YRDTC-MASS*XG)*(NVDTC-MASS*XG)
      BC=-((MASS-YVDTC)*(NRC-MASS*XG*UO)+YVC*(IZ-NRDTC)+NVC*(YRDTC-...
        MASS*XG)+(YRC-MASS*UO)*(NVDTC-MASS*XG))
      CC=YVC*(NRC-MASS*XG*UO)-NVC*(YRC-MASS*UO)
      SGMAC1=(-BC/AC+SQRT((BC/AC)**2-4.*CC/AC))*0.5
      WRITE(6,1500) XF,SGMAC1
 1500 FORMAT(5X,'XF=',F5.3,5X,'SIGMA=',F10.5)
DERIVATIVE
      TMODEL=TIME*LMODEL/UMODEL
      TSHIP=TIME*LSHIP/USHIP
      D=-DRATE*(RAMP(TLAG)-RAMP(TDMAX/DRATE))/57.3
      U=INTGRL(UO,UDOT)
      VC=INTGRL(VO,VDOTC)
      RC=INTGRL(RO,RDOTC)
      PSIC=INTGRL(PSIO,RC)
      F2C=YO+YVC*VC+1./6.*YVVV*VC**3+1./2.*YVRR*VC*RC**2+(YRC-MASS)*...
          RC+1./6.*YRRR*RC**3+.5*YRVV*RC*VC**2+YD*D
      F3C=NO+NVC*VC+1./6.*NVVV*VC**3+.5*NVRR*VC*RC**2+(NRC-MASS*XG)...
          *RC+1./6.*NRRR*RC**3+NRVV*RC*VC**2*.5+NDP*D
      NMVDTC=(IZ-NRDTC)*F2C-(MASS*XG-YRDTC)*F3C
      NMRDTC=(MASS-YVDTC)*F3C-(MASS*XG-NVDTC)*F2C
      VDOTC=NMVDTC/DETC
      RDOTC=NMRDTC/DETC
      XAXDTC=U*COS(PSIC)-VC*SIN(PSIC)
      YAXDTC=U*SIN(PSIC)+VC*COS(PSIC)
      XAXC=INTGRL(XAXO,XAXDTC)
      YAXC=INTGRL(YAXO,YAXDTC)
SAMPLE
      NUM=NUM+1
      CALL DRWG(1,1,NUM,YAXC,XAXC)
CONTRL FINTIM=29.,DELT=0.01,DELS=0.02
```

```
PRINT .50,XAXC,YAXC,PSIC,VC,RC,TMCDEL
TERMINAL
     CALL ENDRW(NPLOT)
     GO TO(1,2,3,4,5,6,7,8),NPLOT
   8 XF=0.38
     GO TO 11
   7 XF=0.36
     GO TO 11
   6 XF=0.34
     GO TO 11
   5 XF=0.32
     GO TO 11
   4 XF=0.31
     GC TO 11
   3 XF=0.30
     GO TO 11
   2 XF=0.28
     GO TO 11
   1 XF=0.27
  11 CALL RERUN
END
STCP
//PLOT.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//      DCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLCT.SYSIN DD *
TEST CF RUDDER POSITION ON THE MODEL A
D.S.PARK    061115
```

< COMPUTER PROGRAM #7 >

```
*** RESPONSES OF THE MODEL A,B,C,D AND E TO  A  SMALL DISTURBANCE ***

* NOTE; SYMBOLS OF VECTOR ARRAY;
*       I=1;MODEL A, 2;MLDEL B, 3;MODEL C, 4;MODEL D, 5;MODEL E


//PARK1857 JOB (2595,0286FT,NH22),'PARK',TIME=4
// EXEC DSL4,REGION=150K
//DSL.INPUT DD *
LABEL TFE RESPONSE OF THE MODELS TO SMALL DISTURBANCE
INTGER NUM,NPLOT,I
INTEG RKSFX
CONST NPLOT=1
CONST DRAFT=.267,LBP=5.,CDH=.0190,YADMAS=.18,XP=.011,MASS=.2,K1=.022,...
      XBAR=.024,X0=.307,ZADMAS=.165,IZ=.0125,XG=-.015,U0=1.
STORAG YV(5),NV(5),YR(5),NR(5),YVDT(5),NVDT(5),YRDT(5),NRDT(5),...
      A(5),B(5),C(5),S1(5),S2(5),V(5)
TABLE YV   (1-5)=-0.35435,-0.31306,-0.31306,-0.31306,-0.18668,...
      NV   (1-5)=-0.09382,-0.11446,-0.12657,-0.14474,-0.17765,...
      YR   (1-5)=+0.07738,+0.05674,+0.04463,+0.02646,-0.00645,...
      NR   (1-5)=-0.06347,-0.05315,-0.04346,-0.03801,-0.02155,...
      YVDT (1-5)=-0.18797,-0.18675,-0.18675,-0.18675,-0.18000,...
      NVDT (1-5)=+0.00399,+0.00377,+0.00243,+0.00102,+0.00000,...
      YRDT (1-5)=+0.00399,+0.00377,+0.00243,+0.00102,+0.00000,...
      NRDT (1-5)=-0.01319,-0.01289,-0.01213,-0.01171,-0.01120
INITIAL
      NUM=0
      WRITE(6,2000)
 2000 FORMAT(4X,'TIME',8X,'V1',4X,'V2',4X,'V3',4X,'V4',4X,'V5')
DYNAMIC
      DO 1000 I=1,5
      A(I)=(MASS-YVDT(I))*(IZ-NRDT(I))-(YRDT(I)-MASS*XG)*(NVDT(I)-...
          MASS*XG)
      B(I)=-((MASS-YVDT(I))*(NR(I)-MASS*XG*UO)+YV(I)*(IZ-NRDT(I))+...
          NV(I)*(YRDT(I)-MASS*XG)+(YR(I)-MASS*UO)*(NVDT(I)-MASS*XG))
      C(I)=YV(I)*(NR(I)-MASS*XG*UO)-NV(I)*(YR(I)-MASS*UO)
        S1(I)=(-B(I)/A(I)+SQRT((B(I)/A(I))**2-4.*C(I)/A(I)))*0.5
        S2(I)=(-B(I)/A(I)-SQRT((B(I)/A(I))**2-4.*C(I)/A(I)))*0.5
      V(I)=EXP(S1(I)*TIME)-EXP(S2(I)*TIME)
 1000 CONTINUE
      WRITE(6,3000) TIME,V
 3000 FORMAT(2X,F5.3,4X,5F10.5)
CONTRL FINTIM=10.,DELT=0.01,DELS=0.01
SAMPLE
      NUM=NUM+1
      V3=V(3)*0.1
```

```
      V4=V(4)*0.005
      V5=V(5)*0.000001
      CALL DRWG(1,1,NUM,TIME,V(1))
      CALL DRWG(1,2,NUM,TIME,V(2))
      CALL DRWG(1,3,NUM,TIME,V3)
      CALL DRWG(1,4,NUM,TIME,V4)
      CALL DRWG(1,5,NUM,TIME,V5)
PRINT S1
TERMINAL
      CALL ENDRW(NPLOT)
END
STOP
//PLOT.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//     DCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT.SYSIN DD *

                                        D.S.PARK                             05
0.        1.5       0.        0.3       7.        6.
```

< COMPUTER PROGRAM #8 >

```
***ZIG-ZAG MANEUVERS OF MODEL A AND C***

//PARK1858 JOB (2595,0286FT,NH22),'PARK',TIME=8
//  EXEC DSL4,REGION=150K
//DSL.INPUT DD *
INTGER NUM,NPLOT
INTEG RKSFX
CONST NPLOT=1
*
*ARBITRARY INITIAL TIME FOR RAMP FUNCTION
*
CONST TA1=0.,TA2=1000.,TA3=1000.,TA4=1000.,TA5=1000.,...
      TB1=0.,TB2=1000.,TB3=1000.,TB4=1000.,TB5=1000.,...
      TC1=0.,TC2=1000.,TC3=1000.,TC4=1000.,TC5=1000.,...
      TD1=0.,TD2=1000.,TD3=1000.,TD4=1000.,TD5=1000.
CONST YVA=-.35435,NVA=-.09382,YRA=.07738,NRA=-.06347,...
      YVDTA=-.18797,NVDTA=.00399,YRDTA=.00399,NRDTA=-.01319
CONST YVC=-.31306,NVC=-.12657,YRC=.04463,NRC=-.04346,...
      YVDTC=-.18675,NVDTC=.00243,YRDTC=.00243,NRDTC=-.01213
CONST YVVV=-12.9,YRRR=-.2766,YRVV=-.2,NVVV=-2.31,...
      NVRR=-.612,NRRR=-.606,NRVV=-2.84,YVRR=-2.36
CONST IZ=.0125,MASS=.2,XG=-.015,Y0=.00016,N0=-.0003,...
      ND=-.03293,YD=.06585,U0=1.,V0=0.,R0=0.,XAX0=0.,YAX0=0.
INITIAL
      NUM=0
      U=1.
      XUDOT=0.025
      PSI0=0.0
      DMAX=20./57.3
      F1=0.
      DETA=(MASS-YVDTA)*(IZ-NRDTA)-(MASS*XG-NVDTA)*(MASS*XG-YRDTA)
      DETC=(MASS-YVDTC)*(IZ-NRDTC)-(MASS*XG-NVDTC)*(MASS*XG-YRDTC)
      DRATE=40./57.3
DERIVATIVE
      DA=DRATE*(RAMP(TA1)-RAMP(TA2)-RAMP(TA3)+RAMP(TA4)+RAMP(TA5))
      DC=DRATE*(RAMP(TC1)-RAMP(TC2)-RAMP(TC3)+RAMP(TC4)+RAMP(TC5))
      VA=INTGRL(V0,VDOTA)
      RA=INTGRL(R0,RDOTA)
      PSIA=INTGRL(PSI0,RA)
      F2A=Y0+YVA*VA+1./6.*YVVV*VA**3+1./2.*YVRR*VA*RA**2+(YRA-MASS)*...
           RA+1./6.*YRRR*RA**3+.5*YRVV*RA*VA**2+YD*DA
      F3A=N0+NVA*VA+1./6.*NVVV*VA**3+.5*NVRR*VA*RA**2+(NRA-MASS*XG)...
           *RA+1./6.*NRRR*RA**3+NRVV*RA*VA**2*.5+ND*DA
      NMVDTA=(IZ-NRDTA)*F2A-(MASS*XG-YRDTA)*F3A
      NMRDTA=(MASS-YVDTA)*F3A-(MASS*XG-NVDTA)*F2A
```

```
      VDOTA=NMVDTA/DETA
      RDOTA=NMRDTA/DETA
      XAXDTA=U*COS(PSIA)-VA*SIN(PSIA)
      YAXDTA=U*SIN(PSIA)+VA*COS(PSIA)
      XAXA=INTGRL(XAXO,XAXDTA)
      YAXA=INTGRL(YAXO,YAXDTA)
      VC=INTGRL(VO,VDOTC)
      RC=INTGRL(RO,RDOTC)
      PSIC=INTGRL(PSIO,RC)
      F2C=YO+YVC*VC+1./6.*YVVV*VC**3+1./2.*YVRR*VC*RC**2+(YRC-MASS)*...
           RC+1./6.*YRRR*RC**3+.5*YRVV*RC*VC**2+YD*DC
      F3C=NO+NVC*VC+1./6.*NVVV*VC**3+.5*NVRR*VC*RC**2+(NRC-MASS*XG)...
           *RC+1./6.*NRRR*RC**3+NRVV*RC*VC**2*.5+ND*DC
      NMVDTC=(IZ-NRDTC)*F2C-(MASS*XG-YRDTC)*F3C
      NMRDTC=(MASS-YVDTC)*F3C-(MASS*XG-NVDTC)*F2C
      VDOTC=NMVDTC/DETC
      RDOTC=NMRDTC/DETC
      XAXDTC=U*COS(PSIC)-VC*SIN(PSIC)
      YAXDTC=U*SIN(PSIC)+VC*COS(PSIC)
      XAXC=INTGRL(XAXO,XAXDTC)
      YAXC=INTGRL(YAXO,YAXDTC)
      PSIAA=-PSIA
      PSICC=-PSIC
DYNAMIC
      IF((TIME.LT.TC3).AND.(ABS(DMAX-DC).LT.0.001)) TC2=TIME
      IF((TIME.LT.TA3).AND.(ABS(DMAX-DA).LT.0.001)) TA2=TIME
      IF((DC.GT.0.0).AND.(ABS(DC+PSIC).LT.0.001)) TC3=TIME
      IF((DA.GT.0.0).AND.(ABS(DA+PSIA).LT.0.001)) TA3=TIME
      IF((TIME.LT.TC5).AND.(ABS(DMAX+DC).LT.0.001)) TC4=TIME
      IF((TIME.LT.TA5).AND.(ABS(DMAX+DA).LT.0.001)) TA4=TIME
      IF((DC.LT.0.0).AND.(ABS(DC+PSIC).LT.0.001)) TC5=TIME
      IF((DA.LT.0.0).AND.(ABS(DA+PSIA).LT.0.001)) TA5=TIME
      IF((TIME.GT.TC5).AND.(ABS(DC).LT.0.001)) TC1=TIME
      IF((TIME.GT.TA5).AND.(ABS(DA).LT.0.001)) TA1=TIME
      IF(TA2.LT.TA1) GO TO 1500
      IF(TC2.GT.TC1) RETURN
      TC2=1000.
      TC3=1000
      TC4=1000.
      TC5=1000.
      RETURN
 1500 TA2=1000.
      TA3=1000.
      TA4=1000.
      TA5=1000.
SAMPLE
      NUM=NUM+1
      CALL DRWG(1,1,NUM,TIME,DA)
```

```
      CALL DRWG(1,2,NUM,TIME,PSIAA)
      CALL DRWG(1,3,NUM,TIME,DC)
      CALL DRWG(1,4,NUM,TIME,PSICC)
      CALL DRWG(2,1,NUM,TIME,VA)
      CALL DRWG(2,2,NUM,TIME,RA)
      CALL DRWG(2,3,NUM,TIME,VC)
      CALL DRWG(2,4,NUM,TIME,RC)
      CALL DRWG(3,1,NUM,XAXA,YAXA)
      CALL DRWG(3,2,NUM,XAXC,YAXC)
CONTRL FINTIM=17.,DELT=0.005,DELS=0.02
PRINT .05,DA,DC,PSIA,PSIC,TA1,TA2,TA3,TA4,TA5
TERMINAL
      CALL ENDRW(NPLOT)
END
STOP
//PLOT.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//      DCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLCT.SYSIN DD *

                                        D.S.PARK                              05
0.0       4.        -1.       .4        5.        6.

                                        D.S.PARK                              05
0.0       4.        -0.6      .3        5.        6.

                                        D.S.PARK                              05
0.        3.        -3.       1.5       6.        5.
```

< COMPUTER PROGRAM #9 >

```
***  SPIRAL MANEUER OF MODEL C  ***

//PARK1859 JOB (2595,0286FT,NH22),'PARK',TIME=4
// EXEC DSL4,REGION=150K
//DSL.INPUT DD *
LABEL SPIRAL MANEUVERING
INTGER NUM,NPLOT
INTEG RKSFX
CONST NPLOT=1
CONST YVC=-.31306,NVC=-.12657,YRC=.04463,NRC=-.04346,...
      YVDTC=-.18675,NVDTC=.00243,YRDTC=.00243,NRDTC=-.01213
CONST YVVV=-12.9,YRRR=-.2766,YRVV=-.2,NVVV=-2.31,...
      NVRR=-.612,NRRR=-.606,NRVV=-2.84,YVRR=-2.36
CONST IZ=.0125,MASS=.2,XG=-.015,Y0=.00016,N0=-.0003,...
      ND=-.03293,YD=.06585,U0=1.,V0=0.,R0=0.,XAXO=0.,YAXO=0.
CONST TT=1000.
INITIAL
      NUM=0
      RC1=0.00
      D=-10./57.3
      DETC=(MASS-YVDTC)*(IZ-NRDTC)-(MASS*XG-NVDTC)*(MASS*XG-YRDTC)
      PSIO=0.0
      WRITE(6,1500)
 1500 FORMAT(51X,'RUDDER ANGLE',4X,'YAW RATE',/,51X,'(DEGREE)',4X,
     1'(RAD./SEC*L/U)')
DERIVATIVE
      VC=INTGRL(VO,VDOTC)
      RC=INTGRL(RO,RDOTC)
      PSIC=INTGRL(PSIO,RC)
      F2C=YO+YVC*VC+1./6.*YVVV*VC**3+1./2.*YVRR*VC*RC**2+(YRC-MASS)*...
            RC+1./6.*YRRR*RC**3+.5*YRVV*RC*VC**2+YD*D
      F3C=NO+NVC*VC+1./6.*NVVV*VC**3+.5*NVRR*VC*RC**2+(NRC-MASS*XG)...
            *RC+1./6.*NRRR*RC**3+NRVV*RC*VC**2*.5+ND*D
      NMVDTC=(IZ-NRDTC)*F2C-(MASS*XG-YRDTC)*F3C
      NMRDTC=(MASS-YVDTC)*F3C-(MASS*XG-NVDTC)*F2C
      VDOTC=NMVDTC/DETC
      RDOTC=NMRDTC/DETC
DYNAMIC
      IF(ABS(RDOTC).GT.0.00001) RETURN
      RC1=RC
      DEGREE=D*57.3
      WRITE(6,1000) DEGREE,RC1
 1000 FORMAT(53X,1F6.3,7X,1F15.8)
      IF(TIME.GT.TT) GO TO 2000
      IF(D.GT.+10/57.3) TT=TIME
```

```
      D=D+0.017452
      RETURN
 2000 D=D-0.017452
PRINT RC
SAMPLE
      NUM=NUM+1
      CALL DRWG(1,1,NUM,D,RC1)
CONTRL FINTIM=425.,DELT=0.01,DELS=0.25
TERMINAL
      CALL ENDRW(NPLOT)
END
STOP
//PLOT.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(3,1)),
//      DCB=(RECFM=FBA,LRECL=37,BLKSIZE=3589)
//PLOT.SYSIN DD *

                                          D.S.PARK                        120005
-0.5      0.2       -0.6      0.2         5.        6.
```

LIST OF REFERENCES

1. Principles of Naval Architecture, SNAME, 1968

2. M. A. Abkowitz, Stability and Motion Control of Ocean Vehicles, M.I.T. 1969.

3. L. I. Schiff and M. Gimprich, "Automatic Steering Ships by Proportional Control", Transaction, v. 57, SNAME.

4. N. H. Norrbin, A Study of Course Keeping and Maneuvering Performance First Symposium on Ship Maneuverability, Dept. of Navy, 1960.

5. Beer and Johnston, Mechanics for Engineering, McGraw-Hill, 1970.

6. Haruzo Eda and C. L. Crane, Jr., "Steering Characteristics of Ships in Calm Water and Waves", Transaction, v. 73 1965.

7. Thaler and Brown, Analysis and Design of Feedback Control Systems, McGraw-Hill, 1960.

8. H. H. Chen, "Some Aspects of Ship Maneuverability", Journal of Ship Research, v. 13, 1969.

9. N. Minorsky, "Directional Stability of Automatically Steered Bodies", Naval Engineering, 1922.

10. DSL/360 User Manual, I.B.M., 1968.

11. NSRDC Report 2130, A Digital Computer Technique for Prediction of Standard Maneuvers of Surface Ships, by J. Storm-Tejsen.

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The factors affecting the stability and the maneuverability of a dynamically unstable ship were studied using the linear and non-linear equations of the motion of the ship. The DSL/360 Language was used to simulate the dynamics and to study both unstable motions and standard maneuvers.

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